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THE GAUGERS GUIDE

AND MEASURER'S MANUAL

THE

GAUGER'S GUIDE

AND

MEASURER'S MANUAL,

CONTAINING

CASK GAUGING

IN ALL ITS BRANCHES;

MALT GAUGING AND TIMBER MEASURING,

WITH A MINUTE DESCRIPTION OF THE

CALLIPERS AND REQUISITE SLIDE-RULES;

Accompanied with Tables of Areas for the Imperial Gallon and Bushel.

BY

THOMAS KENTISH,

Author of "A Treatise on a Box of Mathematical Instruments and the Slide-rule, Geometry, Mensuration, Trigonometry, and Navigation,"
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"Emblematical and Statistical Map of England and Wales,"
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PREFACE.

In the second edition of my Treatise on the Slide-rule, I expressed a hope that I had interspersed it with many remarks calculated to abbreviate the labour of computation; and so modified the rules for surfaces and solids generally, as to embrace almost every species of mensuration under the simplest form. As I have seen the whole of what was original transferred, with a liberal hand, to the pages of other writers, and their modes of solution, in several instances, entirely altered, in conformity with my suggestions, I presume the information was deemed worth copying.

In the present work I have, of course, introduced it myself; and having subsequently paid considerable attention to the different varieties of Slide-rule, I have been enabled, occasionally, to devise still more compendious methods of operation; and to extend, in some slight degree, the uses of an instrument, overlooked by the Mathematician generally, but duly valued by the Revenue Officer, and worthy of his warmest appreciation.

As any knowledge of it would be imperfect without a thorough comprehension of its construction, I have entered into a minute description of the mode of laying down the various lines graduated upon it; and trust they will be found so clearly elucidated, that no one can possibly fail in perfectly understanding them.

The principle of the inverted line has never, so far as I am aware, been at all satisfactorily explained. It is, however, one of the most useful in the whole range of the Slide-rule. I have made it my especial study; and think, after the explanation I have given of it, it will be found that little more remains to be said upon the subject.

On the spheroid and variety lines I have also been particularly explicit. The calculations for the former are simple, and for the latter they are inserted; so that the student will be able to construct or verify them without difficulty.

With respect to the ullage lines, it is probable they were originally constructed by the tedious process of dipping. After many attempts to devise a better plan of illustration, it at last occurred to me, that, by a modification of the Tell-tale, screwed into the under part of the cask, so as to descend to a lower level than the bottom, the desired object would be effected with a facility and precision not to be obtained by any other method. This exhibits them so clearly, that they will be understood by inspection. It has also enabled me to arrange the numbers for calculation in so easy and natural a manner, that the reason for the mode of solution will be comprehended without difficulty, and recollected without hesitation.

To the excellent rule devised by Thomas Simpson, for approximating the computation of areas by equidistant parallel lines, as well as to the corresponding one for estimating the capacities of solids by equidistant parallel planes, especial prominence has been given; and, that they may be carefully treasured in remembrance, I have ventured to introduce a plan, adopted primarily for self-assistance, of marking the diagrams with emblems, which, conveying to the mind ideas through the eye, will be found to make not only an immediate but a permanent impression upon the memory; an object I have endeavoured particularly to effect, as knowledge, to be practically useful, must be at the moment accessible, so that we may be enabled to proceed without error or hesitation.

The multiplication of duodecimals has been a subject of misconception to many persons, and a source of perplexity to Mathematicians themselves, as is evident from the various and conflicting attempts that have been made to elucidate them. They have formed, at once, the easiest and the most embarrassing portion of arithmetic. My description of them will be found, as far as I am aware, altogether different from any hitherto given; but I flatter myself it is the true one; and that I have, at length, fortunately, entirely explained away the apparent difficulty attending them.

Having previously experienced much inconvenience in attempting to distinguish divisors from gauge points, I have, throughout the present work, annexed the quadratic index to the latter, which has enabled me to arrange the formulæ in a much neater method than before. This plan explains itself, and is not attended with uncertainty.

A few abbreviations, long made use of in private, have been introduced where thought desirable; as well as some half dozen new terms, which I have ventured to form, where I found none in the language suited to my purpose. They have been adopted, not for the sake of novelty, which, in itself, is no recommendation, but for the purpose of expressing in one word what would otherwise have required the employment of many.

London, May 4th, 1861.

EXPLANATION OF CHARACTERS EMPLOYED.

= **Equal.** The sign of equality, as 277.274 cubic inches = 1 imperial gallon.

+ **Plus, or More.** The sign of addition, as $12 + 4 = 16$; 12 plus 4 equals 16.

— **Minus, or Less.** The sign of subtraction, as $12 - 4 = 8$; 12 minus 4 equals 8.

× **Into, or Multiplied by.** The sign of multiplication, as $12 \times 4 = 48$; 12 multiplied by 4 equals 48.

In Algebra a dot is often employed instead, the expression $3\,x\,.\,2\,m$ being the same as $3\,x \times 2\,m$—that is, thrice x multiplied by twice m; and 4 . diff.^{2} the same as 4 × diff.^{2}, viz., 4 times the square of the difference.

$\div$ Divided by. The sign of division, as $12 \div 4 = 3$; 12 divided by 4 equals 3.

This abbreviation may be considered as the representative of a vulgar fraction; the dots are therefore often omitted, and the dividend is placed above the line, and the divisor below, instead; so that the expressions $12 \div 4$ and $\frac{12}{4}$ are equivalent.

The lower figure is termed the denominator, that is, the *name* of the fractional part; the upper figure is called the numerator, because it *enumerates*, or shows the *number* of the equal parts: thus $\frac{3}{7}$ denotes that we divide a thing into seven equal parts, hence called *sevenths;* and take 3 of them, 3 sevenths.

$: \;::\; :$ The signs of proportion, as $2 : 4 :: 8 : 16$, which is read 2 are to 4 as 8 are to 16; or, as 2 are to 4, so are 8 to 16.

$\propto$ Varies as. Thus $A \propto B$ is read, A varies as B.

$\therefore$ Consequently, or Therefore. Thus $\frac{3}{5} = 8$ $\therefore \frac{1}{5} = 2\frac{2}{3} \therefore \frac{5}{5} = 10\frac{10}{3} = 13\frac{1}{3}$ is read, 3 fifths of a number equal 8; consequently, 1 fifth of it equals $2\frac{2}{3}$; consequently, 5 fifths, or the whole number, equal 10 and 10 thirds, which are equal to 13 and 1 third.

$\overline{\quad}$ and ( ). A vinculum or bar, and a parenthesis, are employed to denote that all the quantities under the vinculum, or included within the parenthesis, are to be considered as one quantity, as $12 - \overline{4 + 3}$, or $12 - (4 + 3) = 5$; that is, 12 minus the sum of 4 and 3 equals 5. If the vinculum or parenthesis be omitted, the expression is altogether different, for $12 - 4 + 3 = 11$.

^{2} The quadratic index, or sign of the square, or second power, as $3^2 = 9$.

^{3} The cubic index, or sign of the cube, or third power, as $3^3 = 27$.

$\surd$ The quadratic radical, or sign of the square root, as $\sqrt{9} = 3$.

$\sqrt[3]{}$ The cubic radical, or sign of the cube root, as $\sqrt[3]{27} = 3$.

When the index, or radical, is applied to more terms than one, the whole of such terms are included within a parenthesis, or placed under a vinculum; and the vinculum, when used with the index, is generally bent at the end into the form of a hook, as $(2\,x)^2 = 4\,x^2$, or $\overline{2\,x}\rceil^2 = 4\,x^2$; $(3\,x)^3 = 27\,x^3$, or $\overline{3\,x}\rceil^3 = 27\,x^3$; $\sqrt{4\,x^2} = 2\,x$; $\sqrt[3]{27\,x^3} = 3\,x$.

When applied to fractions, the stem of the radical and the hook of the vinculum are elongated, if intended to include both numerator and denominator, as $\sqrt{\frac{4}{9}} = \frac{2}{3}$;

$$\left.\overline{\frac{3\,x}{4}}\right|^{3} = \frac{27\,x^{3}}{64}$$

In the present work I have extended this principle to include several terms under one index, or radical—thus,

$$\left.\overline{\begin{array}{r}12\\14\\20\\24\\25\end{array}}\right|^{2} = \begin{array}{r}144\\196\\400\\576\\625\end{array} \qquad \sqrt{\begin{array}{r}144\\196\\400\\576\\625\end{array}} = \begin{array}{r}12\\14\\20\\24\\25\end{array}$$

The mean proportional between two numbers is the square root of their product; that is, the mean proportional between 3 and 12 is 6, viz., $\sqrt{3 \times 12} = 6$. In order, therefore, to exhibit this mean proportional as again squared, for the purpose of taking it on the D line, I have found it necessary to combine the radical and index; thus, $\sqrt{3 \times 12} = 6 \therefore 6^{2} = \left.\sqrt{3 \times 12}\right|^{2}$.

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ERRATA.

Page 99, line 2. Erase *m*, and insert *n*, viz., Read, Through *n* draw *m r*.

Page 154, line 7. In first and second formulæ, for $\overline{2 \delta}$ read $\overline{2 \delta}^2$.

Page 250, line 1. Read, May be known to be the frustum of a perfect spheroid.

THE

GAUGER'S GUIDE

AND

MEASURER'S MANUAL.

LOGARITHMS.

1. When, in a series, numbers increase by equal differences, they are said to be in Arithmetical Progression; as

0	1	2	3	4	5
0	2	4	6	8	10

In the first of these series each number is 1 more than its antecedent; in the second, 2 more. These numbers are in Arithmetical Progression.

2. When, in a series, numbers increase by equal ratios, they are said to be in Geometrical Progression, as

1	2	4	8	16	32
1	3	9	27	81	243

In the first of these series each number is twice its antecedent; in the second, thrice. These numbers are in Geometrical Progression.

3. Logarithms are a series of numbers in Arithmetical Progression, corresponding to another series of numbers in Geometrical Progression: thus

0	1	2	3	4	5	6	7	8
1	2	4	8	16	32	64	128	256

where the indices 0, 1, 2, 3, &c., in the arithmetical series, are the logarithms of the numbers 1, 2, 4, 8, &c., of the geometrical.

4. On examining these, it will be found that if any two of the logarithms, or indices, are added together, their *sum* will be the logarithm, or index, of the *product* of the numbers to which they belong.

Thus 2 and 3 are 5; the number under 2 is 4; the number under 3 is 8; 4 times 8 are 32, which is the number under the index 5.

5. In like manner, if any one of the indices be subtracted from another, their *difference* will be the index of the *quotient* of these numbers.

Thus 5 from 7 leaves 2; the number under 5 is 32; the number under 7 is 128; 32 in 128 will go 4 times, which is the number under the index 2.

6. For the same reason, if any one of the indices be multiplied by another denoting any power, the *product* will be the index of that *power*.

Thus, to find the square of 8, its index is 3; which, doubled, becomes 6, the index of 64, the square of 8, as required.

7. Lastly, if the index of any number be divided by the index of any root, the *quotient* will be the index of that *root*.

Thus, to find the cube root of 64; its index is 6, which, divided by 3, becomes 2, the index of 4, the cube root of 64, as required.

8. From which it appears that addition of logarithms answers to multiplication of common numbers; subtraction to division; multiplication to involution; and division to evolution.

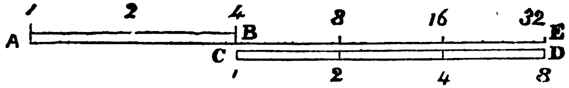
9. The same will also be the case if we select any other geometrical series: thus,

0	1	2	3	4	5
1	4	16	64	256	1024

0	1	2	3	4	5
1	10	100	1000	10000	100000

10. From which it is evident that the same indices may serve for any system; and, consequently, that the varieties of systems of logarithms are infinite.

11. If to a scale of equal parts we attach the geometric series, or continued proportionals, 1, 2, 4, 8, 16, 32, &c., then the distances, increasing by equal differences, will be the logarithms, or logarithmic distances, of the numbers attached to them;—



that is, the distance from A to B will be the logarithm, or logarithmic distance, of the number 4; and the distance from C to D, the logarithm, or logarithmic distance, of the number 8; and, consequently, the *sum* of these distances will represent the *product* of the numbers, and the *difference* of the distances the *quotient* of the numbers—that is,

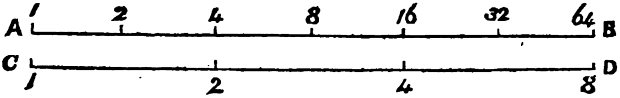
$$AB + CD = 4 \times 8 = AE = 32$$

And $$AE - CD = 32 \div 8 = AB = 4$$

and the numbers on the upper line run in proportion with those on the lower,

$$4 : 1 :: 8 : 2 :: 16 : 4 :: 32 : 8$$

12. As it has been shown that twice the index of any number is the index of the square of that number, and that half the index of any number is the index of the square root of that number, it follows, that if we lay down two scales, in one of which the logarithmic distances shall be double those of the other, we shall then have a series of roots and powers: thus,



Let A B and C D be two lines of equal length, and A B be divided into twice as many equal parts as C D, and attach the continued proportionals 1, 2, 4, 8, &c., as before, then over any number on C D will be its square on A B, and under any number on A B will be its square root on C D.

13. It will be perceived that as the numbers rapidly increase, the logarithmic distances will be proportionally diminished; for somewhere in the space between 2 and 4 the number 3 will have to be inserted; and, in the space between 4 and 8, the numbers 5, 6, and 7.

14. To ascertain the points at which these numbers are to be placed, recourse must be had to a Table of Logarithms.

Referring to such Table, we find log. 1 = 0; log. 2 = 301; log. 3 = 477; log. 4 = 602; log. 5 = 698; log. 6 = 778; log. 7 = 845; log. 8 = 903; log. 9 = 954; and log. 10 = 1000.

15. If, then, by means of a sector and pair of compasses, a diagonal scale, or minutely-divided feather-edged plotting scale, we lay down these distances, and attach to them the appropriate numbers, we shall have the line as graduated on the Slide-rule.

16. Logarithms may be justly considered as among the noblest discoveries of mankind, and one of the most valuable accessions to mathematical science ever made: they were invented by Napier, baron of Mer-

chiston, in Scotland, and published in the reign of James the First, in 1614.

17. About nine years after this the celebrated Gunter conceived the happy idea of laying them down on a scale, by the help of which and a pair of compasses he was enabled to solve questions instrumentally.

18. Oughtred improved upon this plan by laying down the scales in duplicate, and applying one to the other; and Partridge, a little later, graduated the distances on a slide, made to work in a groove, as we have them at the present time. Some have claimed Oughtred to be the inventor, but he himself, in his "Just Apologie against the Slanderous Insimulations of Richard Delamaine," says, "For my circles of proportion I have not so good a claim. The honour of the invention, after Neper and Briggs, belongeth (if I have not been wrongly informed) to Master Gunter, *who exposed their numbers upon a straight line,* which, being once done, was there any such masterie to bring the same line into a circle?" Probably, as Gunter left it, it was, as Oughtred styled it to Forster, "a poore instrument." Any one who has really attempted to lay down a logarithmic line knows how extremely difficult it is to make it perfectly correct: every division may appear to be as accurate as possible, but directly it is moved and placed in juxtaposition with a different part of another line similarly divided, it is seen that the subdivisions do not present the correct-

ness that was expected. In the present day, the brass patterns from which the Rules are graduated, are engine-divided, and the greatest nicety is attained. It is probable that much of Oughtred's expressed contempt for Instrumental Computation arose from his being unable, with such lines as he could construct, to obtain results equally accurate with those produced by Numerical Computation. Now, however, the case is very different. The Slide-rule is a splendid instrument, embracing a vast extent of operations; and were our weights, measures, and currency decimal, there would scarcely be a limit to its use. The chief merit, however, subordinate of course to Napier, is due to Gunter, for hundreds can improve where only one can invent.

THE SLIDE-RULE.

19. The Slide-rule is an instrument containing the logarithmic lines we have been describing; they are arranged in different ways, according to the purpose for which they are intended; but the principle in all is the same. The line A is usually laid down twice along the top of the rule—that is, to a double radius; the line D is generally laid down at the bottom of the rule, once in the same space—that is, to a single radius. Between these two lines is a groove for the reception of a slide, which contains two lines,

B and C, graduated, the one along the upper edge, the other along the lower edge; the upper, or B line intended to work with the A line; the lower, or C line, meant to be used with the D line. The B and C lines are merely copies of the A line.

20. In using the Slide-rule, the first thing to be attended to is the valuation of the numbers in the A, B, and C lines. To the numbers in the first division of the line we may assign what values we please; the values of the numbers in the second division will be ten times those of the first. If the first series be reckoned as .01, .02, .03, &c., the second will be .1, .2, .3, &c. The first .1, .2, .3, &c.; the second 1, 2, 3, &c. The first 1, 2, 3, &c.; the second 10, 20, 30, &c. And here it is immaterial whether a number is chosen from the first or second division; but, in ascertaining the squares and square roots of numbers with C and D, in rules in which the D line commences with unity, it will be necessary to observe, that if the number of figures representing the square be odd, it must be selected from the first division of C; if even, from the second division. Thus, to find the square root of 5 or 500, look under the first division of C; for the square root of 50 or 5000, under the second division of C. To find the square roots of decimals, the number of decimals must first be made even, as in common arithmetic; the effective figures may then be treated as whole numbers. Thus

.05, .0005, contain an even number of figures; but the only effective figure in each is 5; the square root will therefore fall under the first division of C. To find the square root of .5 or .005, make the number of figures even, and they become .50 and .0050. In each of these cases the effective figures are 50, and the square root will fall under the second division of C.

21. When the slide B C is laid *evenly* in the groove—that is, when the commencement of A coincides with the commencement of B, the numbers on A are the *same* as the numbers on B; when the slide is in any other position, the numbers on A are *proportional* to the numbers on B.

22. The same is the case with the D line. When the slide lies evenly in—that is, when unity on C stands over unity on D, the numbers on C are the *squares* of those on D; when the slide is in any other position, the numbers on C are *proportional* to the squares of those on D.

23. Thus, let the slide be drawn back to the left, till the 3 of B falls under the 1 of A; then we have a series of continued proportionals, or equivalent fractions; the odd terms, antecedents, or numerators, standing over the even terms, consequents, or denominators, as $\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12} = \frac{5}{15}$, &c.; or 1 : 3 :: 2 : 6 :: 3 : 9 :: 4 : 12 :: 5 : 15, &c.; and, on the C and D

lines, $\frac{3}{1^2} = \frac{12}{2^2} = \frac{27}{3^2} = \frac{48}{4^2} = \frac{75}{5^2}$, &c.; or $3 : 1^2 :: 12 : 2^2 :: 27 : 3^2 :: 48 : 4^2 :: 75 : 5^2$, &c.; where it may be observed that the arbitrary* characters : :: : which are usually understood as the signs of proportion, may also be read as the marks of division and equality, which they are readily convertible into, by drawing a line between the first and last two dots—and connecting, by lines, the middle two dots; thus, $\div = \div$. So that the expressions $1 : 3 :: 2 : 6 :: 3 : 9 :: 4 : 12$; $1 \div 3 = 2 \div 6 = 3 \div 9 = 4 \div 12$; and $\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12}$ are identical; and either may be read indifferently, 1 is to 3, as 2 are to 6, as 3 are to 9, as 4 are to 12; or 1 divided by 3, equals 2 divided by 6, equals 3 divided by 9, equals 4 divided by 12; or 1 third equals 2 sixths, equal 3 ninths, equal 4 twelfths.

24. A careful attention to the two preceding sections will explain every operation. Whenever we require to square a number, we select this number on D, and look against it on C; whenever we wish to

---

* The four middle dots are Oughtred's invention. Between the antecedents and consequents he used one dot, which has since been appropriated as the distinction between whole numbers and decimals; and, in algebra, as the sign of multiplication.

obtain the square root of a number, we select the number on C, and look against it on D. When we neither require to square it, nor to extract the root, the D line does not enter into the operation, and the A and B lines are used indifferently.

25. In the following pages, therefore, whenever a number occurs with the index ^{2} attached to it, it is to be understood that such number is to be taken on the D line; or, in other words, that 2^2, 4^2, and 2 D, 4 D are synonymous; which latter expressions 2 D, 4 D, are not to be read twice D, four times D, as $2x$ and $4x$ are read twice x, and four times x in Algebra; but *two* D, *four* D—that is, 2 *on* D, 4 *on* D—that is, 2 *squared*, 4 *squared.*

26. In many rules half of the D line is laid above, and half below, the slide; this makes no difference, however, in the working: it simply prevents the A, B, C, and D lines being used in conjunction, which is of little importance.

27. It has been shown that the lines in juxtaposition present a series of proportionals. To exhibit such proportionals is, indeed, the sole purpose of the Slide-rule: it is the Instrument of Proportion, the Rule of the Rule of Three. In every operation required to be performed upon it, three numbers are given, to the third of which a proportional is sought, bearing to it the same ratio as the second to the first.

EXAMPLES.

28. Find a fourth proportional to the numbers 2, 3, and 4—that is,

$$2 : 3 :: 4 : ?$$

2 are to 3 as 4 are to what?

$$\text{Or,} \quad \frac{2}{3} = \frac{4}{?}$$

2 divided by 3, equals 4 divided by what?

$$2\,A : 3\,B :: 4\,A : 6\,B$$

$$\text{Or,} \quad 2\,B : 3\,A :: 4\,B : 6\,A$$

viz., to 2 on A set 3 on B, then under 4 on A is 6 on B; or, to 2 on B set 3 on A, then over 4 on B is 6 on A—that is,

A	2	4
B	3	6

Or,

A	3	6
B	2	4

29. Now to solve questions of this kind numerically, we multiply the second and third terms together, and divide the product by the first. To put the above operation into a formula, therefore, it will stand as follows:

$$\frac{3 \times 4}{2}$$

which is to be read, divide by 2, 3 times 4; or

$$2 : 3 :: 4 : ?$$

2 is to 3, as 4 is to what?

30. Particular attention should be paid to this mode of reading the formulæ, as it is the order in which the numbers are to be taken on the Slide-rule. Begin with the denominator as the first term, and proceed with the numerators as second and third terms.*

31. Find a fourth proportional to the numbers 2^2, 3, and 4^2—that is,

$$2^2 : 3 :: 4^2 : ?$$

$$2\,D : 3\,C :: 4\,D : 12\,C$$

viz., over 2 on D set 3 on C, then over 4 on D is 12 on C.

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* It will also, in cases in which it is desired to present the terms of a Proportion as a Formula, be found a saving of time to throw them into the shape of a Formula at once, without the trouble of first writing them as a Proportion, and composing the Formula afterwards. Thus, suppose the proportion to be $3 : 4 :: 6 : 8$, proceed as follows:—Say, 3 (and, while so saying, write down $_3$) are to (and, while so saying, draw a line over the three $\frac{}{3}$) 4 (and, while so saying, write down four $\frac{4}{3}$) as (and, while so saying, attach the mark ×, $\frac{4\ \times}{3}$) 6 (and, while so saying, write down the six, $\frac{4 \times 6}{3}$) are to (and, while so saying, put =, $\frac{4 \times 6}{3} =$) 8 (and, while so saying, write the eight) $\frac{4 \times 6}{3} = 8$; three are to four as six are to eight. This appears tedious in description, but will be found expeditious in practice.

Here, agreeably to the observation in section 25, the numbers 2^2 and 4^2 are taken on the D line—that is,

C	3	12
D	2	4

As a formula it will stand thus:

$$\frac{3 \times 4^2}{2^2}$$

And is to be read, Divide by 2 squared, 3 times 4 squared; or,

$$2^2 : 3 :: 4^2 : ?$$

2 squared is to 3, as 4 squared is to what?

32. Find a fourth proportional to the numbers 2, 3^2, and 8—that is,

$$2 : 3^2 :: 8 : ?^2$$

$$2\,C : 3\,D :: 8\,C : 6\,D$$

Bring the 2 of C over the 3 of D, then the 8 of C will stand over the 6 of D—that is,

C	2	8
D	3	6

As a formula it will stand thus,

$$\sqrt{\frac{3^2 \times 8}{2}}$$

In common arithmetic this would be read: Find the square root of 8 times the square of 3, divided by 2. For the Slide-rule it must be taken in a backward order: Divide by 2, 3 squared times 8, and take the root; or,

$$2 : 3^2 :: 8 : ?^2$$

2 is to 3 squared, as 8 is to what squared?

33. Multiply 3 by 5.

To 3 of A set 1 of B; over 5 of B is 15 on A.

This is simply to divide by 1, 3 times 5—viz.,

$$\frac{3 \times 5}{1}$$

$$\text{Or,}\quad 1 : 3 :: 5 : ?$$

1 is to 3, as 5 is to what?

34. Find the cube of 2.

This is to divide by 1 squared, 2 times 2 squared: viz.,—

$$\frac{2 \times 2^2}{1^2} = ?$$

$$\text{Or,}\quad 1^2 : 2 :: 2^2 : ?$$

1 squared is to 2 as 2 squared is to what?

1 D : 2 C :: 2 D : 8 C

Find the cube of 4.

$$\frac{4 \times 4^2}{1^2} = ?$$

Divide by 1 squared, 4 times 4 squared.

1 D : 4 C :: 4 D : 64 C

Find the cube of 3.35.

$$\frac{3.35 \times 3.35^2}{1^2} = ?$$

1 squared is to 3.35 as 3.35 squared is to what?

1 D : 3.35 C :: 3.35 D : 37.5 C

35. Divide 8 by 2.

Under 8 of A set 2 of B; over 1 of B is 4 on A. This is simply to divide by 2, 8 times 1; or

$$\frac{8 \times 1}{2}$$

Or, 2 : 8 :: 1 : ?

2 is to 8 as 1 is to what?

36. Multiply 3 by 50.

Under 3 of A place 1 of B; over 5 of B is 150. Here the 5 of B being reckoned as 50, the commencement of the slide will be 10; the 3 over it, therefore, becomes 30, and the numbers *run on* 40, 50, 60, &c., up to 150, found standing over the 5 of B.

37. Divide 4 by 8.

Under the 4 in the second division of A place the 8 of B, denominator under numerator, precisely as in Vulgar Fractions. The numbers on A *run back* 3, 2, 1, .9, .8, &c., till .5 is found standing over the commencement of the slide, or 1 of B.

38. Find a mean proportional between 4 and 9.

To ascertain this, we have simply to take the square root of the product—that is,

$$\sqrt{4 \times 9}$$

In rules in which unity on D coincides with unity on A, this may be effected with the A, B C, and D lines conjunctly: thus,

To 4 on A set 1 on the slide, then under 9 of C is 6 on D; which, in reality, is dividing by 1, 4 times 9, and taking the root—that is, as a formula,

$$\sqrt{\frac{4 \times 9}{1}}$$

$$\text{Or,} \quad 1 : 4 :: 9 : ?^2$$

1 is to 4 as 9 is to what squared?

A preferable way, however, is to make use of the C and D lines only; but for this purpose the formula will require a modification; for, in the proportion $1 : 4 :: 9 : ?^2$, the fourth term, or second consequent, as denoted by the index ^{2}, must be taken on D, as mentioned in section 25: it will be requisite, therefore, that the second term, or first consequent, 4, be represented under the same form—that is, with the index ^{2} attached, that *it* likewise may be selected on D. This is readily effected by multiplying it by itself; its antecedent, the first term, of course being multiplied by the same number, to keep up the proportion. We, accordingly, have

$$4 : 4^2 :: 9 : ?^2$$

4 on C is to 4 on D, as 9 on C is to 6 on D; which, as a formula, will stand thus:

$$\sqrt{\frac{4^2 \times 9}{4}}$$

that is, divide by 4, 4 squared times 9, and take the root; or,

4 is to 4 squared as 9 is to what squared?

39. Hence, to find a mean proportional between two numbers with the C and D lines only, set one of the numbers over itself, and look under the other.

40. This finding of a mean proportional is of great utility for operations on the Slide-rule, as by it we reduce parallelograms to squares, and ellipses to circles: a square whose side is 6 inches being equal to a parallelogram 12 inches by 3; and an ellipse, whose axes are 12 and 3, equal to a circle whose diameter is 6.

41. Find the value of $\frac{8(3^2 + 5^2)}{6^2}$.

This is equal to $\frac{8 \times 3^2}{6^2} + \frac{8 \times 5^2}{6^2}$, and therefore must be effected by obtaining the quotients separately, and then adding them together.

Over 6 on D set 8 on C; then over 3 on D is 2 for the first quotient; and over 5 on D is 5.55 for the second quotient.

$$2 + 5.55 = 7.55$$

that is, $$6 \text{ on D} : 8 \text{ on C} :: \begin{array}{l} 3 \text{ on D} : 2 \quad\;\; \text{on C} \\ 5 \text{ on D} : \underline{5.55} \text{ on C} \\ \qquad\qquad \underline{\underline{7.55}} \end{array}$$

For the purposes of the Slide-rule, the following is a much more convenient form :—

$$\frac{8}{6^2}\left\{\begin{matrix}3\\5\end{matrix}\right)^2 \begin{array}{l} = 2 \\ = \underline{5.55} \\ \quad \underline{\underline{7.55}} \end{array} \text{ viz., } \frac{8\text{ C}}{6\text{ D}}\left\{\begin{array}{l}3\text{ D} = 2. \quad\;\; \text{C}\\ 5\text{ D} = \underline{5.55\text{ C}}\\ \qquad\quad\; \underline{\underline{7.55}}\end{array}\right.$$

Divide by 6 squared 8 times (3 squared, 5 squared).

The index ^{2} attached to 6^2 shows that it is to be taken on the D line; over it is to be set the 8 of the C line, exactly as it stands in the formula; then against the 3 and 5, taken on the D line, as the index denotes, will be found 2 and 5.55.

42. Find the value of $\dfrac{40(24^2 + 32 \;\; + 59.5^2)}{46^2}$

$$\frac{40}{46^2}\left\{\begin{matrix}24\\32\\59.5\end{matrix}\right|^2 \begin{array}{l} = 10.9 \\ = 19.36 \\ = \underline{66.9} \\ \quad \underline{\underline{97.16}} \end{array}$$

Divide by 46 squared 40 times (24 squared, 32 squared, 59.5 squared).

43. Find the value of $\frac{56(32^2+39^2+39^2)}{32.54^2}$

$$\frac{56}{32.54^2}\left\{\begin{array}{l} \left.\begin{array}{c}\overline{32}\\39\\—\end{array}\right)^{2}\begin{array}{l}=54.1\\=80.4\\=80.4\end{array}\\ \qquad\qquad\;\; \overline{\;\;214.9}\end{array}\right.$$

44. Find the value of $\frac{28(24^2+32^2+\overline{24+32}|^2)}{46^2}$

$$\frac{28}{46^2}\left\{\begin{array}{l} \left.\begin{array}{c}\overline{24}\\32\\56\end{array}\right)^{2}\begin{array}{r}=\;\;7.62\\=13.55\\=41.5\;\;\end{array}\\ \qquad\qquad\;\; \overline{\;\;62.67}\end{array}\right.$$

45. Find the value of $\frac{20(20^2+24^2+24^2-\frac{1}{10}\text{ of }8^2)}{32.54^2}$

$$\frac{20}{32.54^2}\left\{\begin{array}{c|lr} \overline{20}^{\;2} & = & 7.55\\ 24 & = & 10.87\\ — & = & 10.87\\ & & \overline{29.29}\\ 8 & =1.2;\ \frac{1}{10} & =.12\\ & & \overline{29.17}\end{array}\right.$$

46. It has been seen, that when the slide lies evenly in, the numbers above the slide are the *same* as those on the slide; and, when it lies unevenly in, that the numbers above the slide are *proportional* to the numbers on the slide: also that, when it lies evenly in, the numbers on the slide are the *squares* of the

numbers below the slide; and, when it lies unevenly in, that the numbers on the slide are *proportional* to the squares of the numbers below the slide.

47. Now if we invert the slide, and lay it in, we find, as a matter of course, every operation the reverse of this: division becomes multiplication, and multiplication division; the sign ÷ passes into × ; ratios and formulæ require to be read in a contrary order, ending where they formerly began, and everything, in short, is the reciprocal of what it was before.

48. Let us then *invert* the slide, and lay it evenly in. We now find that the numbers above the slide are *reciprocally the same* as the numbers on the slide; and if it is laid unevenly in, that the numbers above the slide are *reciprocally proportional* to the numbers on the slide; also, that when it lies evenly in, the numbers on the slide are *reciprocally equal* to the squares of the numbers below the slide; and when it lies unevenly in, that the numbers on the slide are *reciprocally proportional* to the squares of the numbers below the slide.

49. Thus, invert the slide, and lay it evenly in; then we have the 2 of the A line standing over the 5 of the slide, and the 5 of the A line over the 2 of the slide—thus, $\frac{2}{5}\cdots\cdots\frac{5}{2}$ where the upper figure of the first is equal to the lower figure of the second, and the upper figure of the second equal to the lower figure of the first—that is, the numbers are *recipro-*

cally equal—equal when crossed over. So with the slide and the D line: the 25 of the slide stands over the 2 of the D, and the 4 of the slide over the 5 of the D—thus: $\frac{25}{2^2}\cdots\cdots\frac{4}{5^2}$, where the upper figure of the first is equal to the lower expression of the second, and the upper figure of the second equal to the lower expression of the first—that is, as before, *reciprocally equal—equal when crossed over*; $25 = 5^2$, $4 = 2^2$.

50. Now let the slide be drawn a little out to the left, till the 8 of the slide falls under the 1 of the A line; then we have the 1 over the 8, and the 2 over the 4—thus, $\frac{1}{8}\cdots\cdots\frac{2}{4}$, where the upper number of the first bears the same ratio to the lower number of the second as the upper number of the second to the lower number of the first—that is, they are *reciprocally proportional—proportional when crossed over, forward and backward,* 1 : 4 :: 2 : 8.

51. So also with the slide and the D line. The 5 of the slide stands over the 4 of the D, and the 20 of the slide over the 2 of the D—thus, $\frac{20}{2^2}\cdots\cdots\frac{5}{4^2}$, where the upper number of the first bears the same ratio to the lower expression of the second as the upper number of the second to the lower expression of the first—that is, as before, *reciprocally proportional*

—*proportional when crossed over, forward and backward,* $20 : 4^2 :: 5 : 2^2$.

52. Consequently, as, when the slide is laid in, in the usual or direct manner, the numbers in juxtaposition present, with the sign of division between them, a series of equivalent fractions—that is, numbers in pairs, whose *quotients* are equal; so, when the slide is laid in, in an inverted order, we are furnished with a series of numbers, standing in pairs, whose *products* are equal, and therefore requiring between them the sign of multiplication. So, instead of writing $\frac{1}{8} \cdots\cdots \frac{2}{4}$ as above, for the sake of explanation, we now write correctly $\begin{matrix}1\\ \times\\ 8\end{matrix} = \begin{matrix}2\\ \times\\ 4\end{matrix}$ that is, 1 multiplied by 8 equals 2 multiplied by 4; and, instead of

$$\frac{20}{2^2} \cdots\cdots \frac{5}{4^2},\ \begin{matrix}20\\ \times\\ 2^2\end{matrix} = \begin{matrix}5\\ \times\\ 4^2\end{matrix}.$$

20 multiplied by 2 squared equals 5 multiplied by 4 squared.

53. It will be obvious, that, while the numbers on A and D increase from left to right, those on the inverted slide increase from right to left; so that, if laid evenly in, and the commencement of the A be reckoned 1, and, consequently, its remote end 100, and the commencement of the slide, or the 1 at the right hand be reckoned 1, then the end of the slide,

or the 1 on the left, will be 100; the 1 of A standing over 100 of Ↄ, and the 100 of A over the 1 of Ↄ, and the following numbers, among others, will be found:

$$\overset{1}{\underset{100}{\times}} = \overset{2}{\underset{50}{\times}} = \overset{3}{\underset{33\frac{1}{3}}{\times}} = \overset{4}{\underset{25}{\times}} = \overset{5}{\underset{20}{\times}} = \overset{10}{\underset{10}{\times}}$$

viz., 1 multiplied by 100 equals 2 multiplied by 50, equals 3 multiplied by $33\frac{1}{3}$, &c.; and on the ᙠ and D lines $\overset{100}{\underset{1^2}{\times}} = \overset{25}{\underset{2^2}{\times}} = \overset{16}{\underset{2.5^2}{\times}} = \overset{4}{\underset{5^2}{\times}}$, &c., viz., 100 multiplied by 1 squared equals 25 multiplied by 2 squared, equals 16 multiplied by 2.5 squared, &c.

This being premised, we may proceed to the solution of questions.

EXAMPLES WITH THE INVERTED SLIDE.

The A *and* Ↄ *Lines.*

54. Find a fourth proportional to the numbers 3, 4, and 6.

$$3 : 4 :: 6 : ?$$

Multiplying extremes and means, we have $4 \times 6 = 3 \times ?$ or, as a formula, $\frac{4 \times 6}{3} = ?$ both of which are to be read alike: 4 times 6 equals 3 times what?

In reading the proportion, therefore, we say: The second multiplied by the third equals the first multiplied by ? and, in the formula, the first numerator multiplied by the second equals the denominator multiplied by ?

$$4 \times 6 = 3 \times ? \text{ or } \begin{array}{ccc} 4 & & 3 \\ \times & = & \times \\ 6 & & ? \end{array}$$

that is, under 4 of A set 6; under 3 is 8.

55. Find the value of $\frac{7 \times 8}{4}$; 7 times 8 = 4 times ?

$$\begin{array}{ccc} 7 & & 4 \\ \times & = & \times \\ 8 & & ? \end{array}$$

Under 7 of A place 8; under 4 is 14.

56. Multiply 8 by 9—that is, $\frac{8 \times 9}{1} = ?$ $8 \times 9 = 1 \times ?$

$$\begin{array}{ccc} 8 & & 1 \\ \times & = & \times \\ 9 & & ? \end{array}$$

Under 8 of A place 9; under 1 is 72.

57. Divide 8 by 4—that is, $\frac{8 \times 1}{4} = ?$ $8 \times 1 = 4 \times ?$

$$\begin{array}{ccc} 8 & & 4 \\ \times & = & \times \\ 1 & & ? \end{array}$$

Under 8 of A place 1; under 4 is 2.

EXAMPLES WITH THE D AND ꓭ LINES.

58. Required the value of $3^2 \times 7$.

As the numbers on D are squares, the note of interrogation, "?," must be placed sometimes above the ×, sometimes below it, according as we wish the result to be a simple numeral or a square.

$$3^2 \times 7 = 1^2 \times ?$$

$$\begin{array}{ccc} 7 & & ? \\ \times & = & \times \\ 3^2 & & 1^2 \end{array}$$

Over 3 of D set 7 of ꓭ; over 1 is 63.

59. Required the value of $9^2 \times 4$.

$$\begin{array}{ccc} 4 & & ? \\ \times & = & \times \\ 9^2 & & 1^2 \end{array}$$

9 squared by 4 equals 1 squared by ?

To estimate this value proceed thus:—Set the right hand 4 of the inverted slide over the 9 of the D. Say, the 4 is 4; run the eye along the slide backwards till you arrive at the next 4, which, of course, is 40, consequently the end of the slide is 100; but, as it does not run back far enough, bring the eye to the right hand or commencement of the slide, which now becomes 100, and the end of the D line becomes 1; the end of each, taking the place of the beginning, and being, in reality, the same as if a couple of Slide-rules were used, the second running to the left of the first; consequently, the right hand 1 of the slide being 100, the next number running back is 200, then 300, till over the 1 of D is found standing 324, the value of $9^2 \times 4$, as required.

60. Required the cube of 3.

$$\frac{3^2 \times 3}{1^2} = ? \qquad \begin{array}{ccc} 3 & & ? \\ \times & = & \times \\ 3^2 & & 1^2 \end{array}$$

3 squared multiplied by 3 = 1 squared by what? Over 3 of D set 3; over 1 is 27.

61. What is the cube of 12?

$$\begin{array}{ccc} 12 & & ? \\ \times & = & \times \\ 12^2 & & 1^2 \end{array}$$

Over 12 set 12; over 1 is 1728.

As the commencement of D is here 10, the square of which is 100, it adds two ciphers to the 12 of the slide, which becomes 1200; a little farther to the left is 1728.

62. Required the value of $\frac{6^2 \times 5}{4^2}$

$$\begin{array}{ccc} 5 & & ? \\ \times & = & \times \\ 6^2 & & 4^2 \end{array} \qquad 6^2 \times 5 = 4^2 \times ?$$

Over 6 of D set 5; over 4 is 11.25.

63. Required the value of $\sqrt{\dfrac{3 \times 8^2}{7}}$

$$3 \times 8^2 = 7 \times ?^2 \qquad \begin{array}{ccc} 3 & & 7 \\ \times & = & \times \\ 8^2 & & ?^2 \end{array}$$

Under 3 place 8 of D; under 7 is 5.23.

64. Required the cube root of 8.

$$8 \times 1^2 = n \times n^2 \qquad \begin{array}{ccc} 8 & & n \\ \times & = & \times \\ 1^2 & & n^2 \end{array}$$

Set the 8 of the slide over the 1 of D, and run the eye along till a number on the slide is found standing over the *same* number on D—viz., 2 over 2.

65. Required the cube root of 7.

$$\begin{array}{ccc} 7 & & n \\ \times & = & \times \\ 1^2 & & n^2 \end{array}$$

Place 7 of the slide over 1 of D; move the eye carefully along, and 1.91 will be found standing over 1.91.

66. Required the cube root of 6.35.

$$6.35 \times 1^2 = n \times n^2 \qquad \begin{array}{ccc} 6.35 & & n \\ \times & = & \times \\ 1^2 & & n^2 \end{array}$$

Set 6.35 of the slide over 1 of D; and 1.85 will be found standing over 1.85.

EXAMPLES WITH THE FOUR LINES, A, Ɔ, ᗺ, AND D.

67. Required the value of $\frac{8 \times 12}{2^2}$.

$$\begin{array}{ccc} 8 & & ? \\ \times & = & \times \\ 12 & & 2^2 \end{array}$$

8 multiplied by 12 equals 2 squared by what? Under 8 of A place 12 of Ɔ; over 2 of D is 24.

68. Required the value of $\sqrt{\frac{8 \times 9}{12}}$

$$8 \times 9 = 12 \times ?^2 \qquad \begin{array}{ccc} 8 & & 12 \\ \times & = & \times \\ 9 & & ?^2 \end{array}$$

8 multiplied by 9 equals 12 times what squared? Under 8 of A place 9 of the slide; under 12 of ᗺ is 2.45 nearly.

69. The investigation of the properties of the inverted slide has prepared us for the consideration of the construction of Special Rules, or Rules adapted to one particular species of computation, and which, in their limited application, are of the greatest utility. It will have been observed, that, in all the operations we have hitherto performed, we have been able to obtain the product of only *two* numbers, one of which may, or may not, have been a square; but as

solids present themselves under the three dimensions of length, breadth, and thickness, all of which frequently differ from each other, it becomes desirable to ascertain whether, by any adjustment of the logarithmic lines, we can obtain by a single operation the product of *three* numbers.

70. Now it is evident, that, as the inverted line increases the distance, and multiplies from right to left, while the direct one performs the same operation from left to right, we have only to combine the two to obtain the effect desired. To lay down a line by the help of logarithmic tables and a scale is, as has before been observed, a task of almost insuperable difficulty. To copy it, however, when once laid down, is the work of only a few minutes.

71. Procure a slip of cardboard, or paper, as long as the rule, and about half an inch broad; invert the rule and lay it before you, so that the A line shall be towards you; lay the slip of cardboard upon it, and along the edge, with a fine pen or pencil, copy the A line in its inverted order; attach the proper numbers; and mark it as V.

72. Now take the rule with the slide laid in, in the direct manner, and draw it back until the 2 of B falls under the 1 of A; take the cardboard with the inverted or V line and lay it under the slide, so as exactly to cover the D line from end to end. It will now be seen that under the 4 of the slide falls the 5

of the inverted line, $\overset{4}{\underset{5}{\times}}$; 4 times 5 equals 10 times what? Running the eye back, over the 10 of the inverted line we find 2 of the slide. 4 times 5 = 10 times 2. Removing the inverted line, the 2 of the slide becomes 20; then, to multiply this by 3, we have only to run the eye along the A line, and under the 3 will be found 60. We have thus effected the operation, $4 \times 5 \times 3 = 60$.

73. Take up the V line again, and lay it as before. 4 times 5 = 10 times what? The answer is 2. The number, therefore, which, *apart from* the V line, we considered as 20, is, when taken *in connection* with the V line, only 2—that is, 20 divided by the number which stands under it. Consequently, under the 3 of A will now be found only 6, and we have effected the operation, $\frac{4 \times 5 \times 3}{10} = 6$.

74. Remove the V line a little back, till the 8 of it falls under the 2 of the slide; now over the 4 of the V will be found 4 on the slide, $\overset{4}{\underset{4}{\times}}$; we have, therefore, 4 times 4 = 8 times what? Consequently, over the 8 we find 2; then, running the eye along, under the 3 of A we find 6. We have thus divided by 8, 4 times 4 times 3—that is, $\frac{4 \times 4 \times 3}{8} = 6$.

75. Hence it appears, that, if, instead of the D line, there be laid down a line similar to A or B, but in an inverted order, and in such a manner as that a certain number, as, for instance, 8 in the last example, shall fall opposite the commencement of the A line; then, if we set against any number on this inverted, or V line, a second number on the slide, and take a third number on A, under the last we shall have the product of the three numbers, divided by that number which falls opposite the commencement of A.

EXAMPLES.

76. Required the value of $\frac{8 \times 12 \times 15}{9}$

Place the 9 of V opposite the 1 of A; draw the slide to the right till over 8 of V stands 12 of the slide; then under 15 of A is 160 on B.

77. Required the value of $\frac{18 \times 20}{144}$; that is, $\frac{18 \times 20 \times 1}{144}$

Place 144 of V opposite the commencement of A; draw the slide to the right till over the 18 of V stands 20 of the slide; then under 1 of A is 2.5 on B.

78. Required the value of $\frac{20 \times 18 \times 16}{1728}$

Place the V line so that 1728 stands opposite the commencement of A; draw out the slide towards the right, or push it back towards the left, till over 20 of V stands 18 of the slide; then under 16 of A is 3.33, the quotient required.

79. Lastly, take a slip of cardboard, or paper, similar to the preceding, and copy upon it the D line, in an inverted order; attach the numerals, and mark it V^2. Then we shall have the same operations as before, with this difference, that the denominator and one of the numerators will be a square number.

80. Required the value of $\frac{5^2 \times 3 \times 8}{7^2}$

Lay the 7 of the V^2 line opposite the commencement of A; draw the slide to the left till over the 5 of the V^2 line stands 3; then under 8 on A will be found 12.2 on the slide.

81. Required the value of $\frac{18 \times 20}{12^2}$; that is, $\frac{1^2 \times 18 \times 20}{12^2}$

Lay the 12 of the V^2 line opposite the commencement of A; over 1 on this V^2 line place 18 of the slide; then under 20 of A will be found 2.5 on the slide.

82. Required the value of $\frac{20^3}{23^2}$; that is, $\frac{20^2 \times 20}{23^2}$; that is, $\frac{20^2 \times 20 \times 1}{23^2}$

Place the 23 of the V^2 line opposite the commencement of A; then over 20 of V^2 place 20 of the slide; and under 1 of A will be found 15.1 on the slide.

83. Required the value of $\frac{17^2}{25^2}$; that is, $\frac{17^2 \times 1 \times 1}{25^2}$

Place the 25 of the V^2 line opposite the commencement of A; over 17 on this V^2 line place 1 of the slide; then under 1 of A will be found .462 on the slide.

84. Required the value of $\frac{1}{9^2}$; that is, $\frac{1^2 \times 1 \times 1}{9^2}$

Place the 9 of the V^2 line opposite the commencement of A; over 1 on this V^2 line place 1 of the slide; then under 1 of A is .0123 on the slide.

85. If, instead of the usual B C slide, a slide E be laid in, and graduated with a treble radius, then we have a series of cubes and cube roots. This line may also be inverted like the A and D lines.

86. By graduating sines and tangents upon another slide, we effect the solution of questions in Trigonometry and Navigation. For a detailed account of the uses of such lines, the reader is referred to the Author's "Treatise on a Box of Instruments and the Slide-rule, Geometry, Mensuration, Trigonometry, and Navigation," and to his "Compound Slide-rules," containing the E, or cube slide, which, in conjunction with the A line, exhibits Kepler's famous proportion between the cubes of planetary distances and the

squares of their periodic times; and the sine and tangent, or trigonometrical slide, which, working to the A and D lines, solves all questions in Plane Trigonometry and Navigation with the greatest facility, exhibiting the sides and angles at a glance.

RATIOS.

87. Measure of extension is of three kinds—lineal, superficial, and solid: lineal for such things as have length only; superficial for those that have length and breadth; and solid where there are length, breadth, and thickness.

88. When lines vary, they vary simply in the ratio of their lengths. Thus, if the diameter of a circle, or the side of a square, A, be thrice the diameter of a circle, or side of a square, B, then the circumference or diagonal of A will be thrice the circumference or diagonal of B, both being lines.

89. When surfaces vary, they vary in the ratio of the product of their length and breadth. Thus, if a parallelogram, C, measures 4 inches by 5, and a parallelogram, D, 8 inches by 7, then C : D :: 4 × 5 : 8 × 7 :: 20 : 56.

90. Hence, when surfaces vary *proportionally*—that is, when the length and breadth increase or diminish in the same ratio, they vary as the *squares* of their like measures. Thus, if the side or diagonal of a square

the side or perimeter of a polygon, the diameter or circumference of a circle, E, be 9 inches, and the side or diagonal of a square, the side or perimeter of a polygon, the diameter or circumference of a circle, F, be 12 inches, then E : F :: 9^2 : 12^2 :: 81 : 144.

91. Hence, also, if of two surfaces, only one of the dimensions varies, while the other remains the same, the surfaces vary only in the ratio of the varying dimension. Thus, if the length of a parallelogram, G, be 12 inches, and its breadth 7 inches, and the length of another, H, be also 12 inches, while its breadth is 9, then G : H :: 7 : 9.

92. When solids vary, they vary in the ratio of the products of their length, breadth, and thickness. Thus, if a block of stone, I, measures 5 feet long, 3 broad, and 2 thick, and a block, K, measures 7 feet long, 4 broad, and 3 thick, then I : K :: $5 \times 3 \times 2$: $7 \times 4 \times 3$ :: 30 : 84.

93. Hence, when solids vary *proportionally*—that is, when the length, breadth, and thickness increase or diminish in the same ratio, they vary as the *cubes* of their like measures. Thus, if the side or diagonal of a cube, or the diameter or circumference of a globe, L, be 7 inches, and the side or diagonal of a cube, or the diameter or circumference of a globe, M, be 9 inches, then L : M :: 7^3 : 9^3 :: 343 : 729.

94. Hence, if of two solids, only two of the dimensions vary proportionally, while the other dimension

remains the same, the solids vary in the ratio of the square of the varying dimension. Thus, if the length of a cylinder, N, be 12 inches, and its diameter 5, and the length of a cylinder, O, be also 12 inches, and its diameter 6, then N : O :: 5^2 : 6^2 :: 25 : 36.

95. Hence, also, if of two solids, only one of the dimensions varies, while the other two remain unaltered, the solids vary only in the ratio of the varying dimension. Thus, if the diameter of a cylinder, P, be 6 inches, and its length 9, and the diameter of a cylinder, Q, be also 6 inches, and its length 10, then P : Q :: 9 : 10.

96. Hence, if there be three globes, and their diameters be in the proportion 2, 5, 9, their surfaces will be as 2^2, 5^2, 9^2, or 4, 25, and 81; and their solidities as 2^3, 5^3, 9^3, or 8, 125, and 729.

97. So, if there be two similar funnels or cones, R and S, and R be filled with water to the depth of 1 foot, and S to the depth of 2 feet, then the circumference of the top of the water in S will be twice that of R, both being lines; the area of the top of the water in S will be 4 times that of R, or 2^2, both being surfaces; and the weight or quantity of water in S will be 8 times that of R, or 2^3, both being solids; and so of all surfaces and solids that vary proportionally.

98. If a number of regular polygons have equal perimeters, *that* contains the greatest amount of sur-

face in which the perimeter is distributed among the greatest number of sides; and, as a circle may be conceived to be a polygon of an *infinite* number of sides, *it* therefore contains the greatest quantity of space within the shortest bounding line.

99. Of isoperimetrical figures, or figures with equal perimeters, a regular polygon contains more than an irregular polygon of the same number of sides. Thus, an equilateral triangle has a greater area than any other triangle of equal ambit; and a square is the largest quadrilateral that can be constructed with sections of the same line.

100. If the three sides of a right-angled triangle be 3, 4, and 5, the area will be 6. If the three sides of an equilateral triangle be each 4, the area will be $4\sqrt{3}$, or 6.9282. The sum of the sides is, in each case, 12, but the area of the latter is nearly $\frac{1}{6}$ more than that of the former.

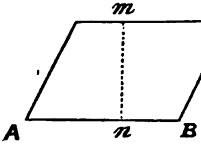
101. If a parallelogram measures 9 inches by 3, its area will be 27. If the 4 sides of a square be each 6, its area will be 36. The sum of the sides is, in each case, 24; but the area of the square is $\frac{1}{3}$ more than that of the parallelogram.

102. In the same way as the circle contains the largest surface within the least compass, so the sphere contains the greatest bulk within the smallest space.

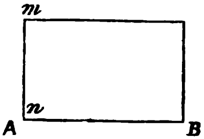
SURFACES AND SOLIDS.

103. A figure containing four sides is called a quadrilateral.

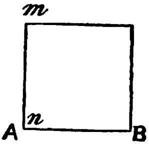
If the opposite sides are parallel, it is a parallelogram.



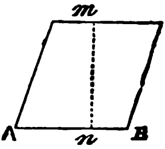
If the angles of the parallelogram are right angles, it is a rectangle.



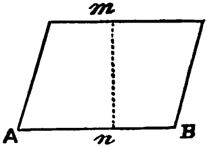
If the rectangle has all its sides equal, it is a square.



If the sides of a parallelogram are equal, but its angles not right angles, it is a rhombus.



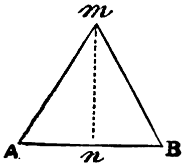
If the opposite sides only are equal, and the angles are not right angles, it is a rhomboid.



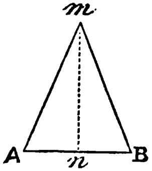
The area of all the preceding figures is found by multiplying the length by the perpendicular breadth —that is, A B *multiplied by m n.*

104. A figure containing three sides is called a triangle.

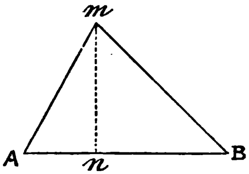
If the three sides are equal, it is an equilateral triangle.



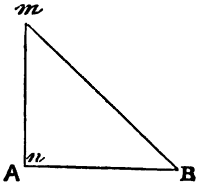
If two sides only are equal, it is an isosceles triangle.



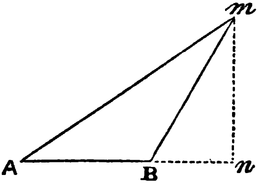
If the three sides are unequal, it is a scalene triangle.



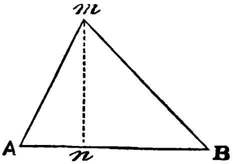
If one of the sides is perpendicular to another, it is a right-angled triangle.



If one of the angles is greater than a right angle, it is an obtuse-angled triangle.

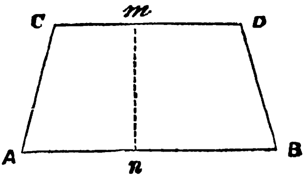


If neither of the angles is equal to a right angle, it is an acute-angled triangle.



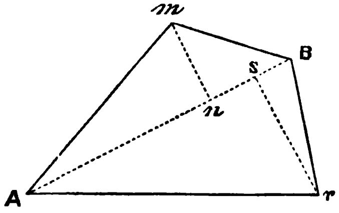
The area of all triangles is found by multiplying the base by the perpendicular height, and taking half the product—that is, A B multiplied by *m n*, and divided by 2.

105. A quadrilateral that has only one pair of its sides parallel is called a trapezoid.



To find its area, multiply the sum of the parallel sides by the perpendicular distance between them, and take half the product—that is, add A B to C D, multiply by *m n*, and divide by 2.

106. A quadrilateral having none of its sides parallel is termed a trapezuim.



To find its area, draw a diagonal from one of its angles to the opposite, and let fall perpendiculars upon it from the other angles. Multiply the sum of the perpendiculars by the diagonal, and take half the product—that is, add *m n* to *s r*, multiply by A B, and divide by 2.

107. Plane figures containing more than four sides are called multilaterals or polygons.

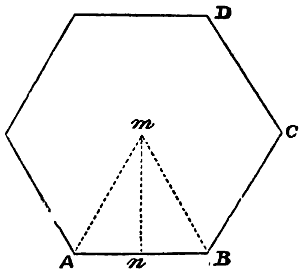
If all the sides are equal, they are called regular polygons; if unequal, irregular polygons.

108. To find the area of an irregular polygon, divide

it into triangles, and compute the area of each separately.

A polygon of three sides is called a trigon; of four, a tetragon; of five, a pentagon; of six, a hexagon; of seven, a heptagon; of eight, an octagon; of nine, a nonagon; of ten, a decagon; of eleven, an undecagon; of twelve, a dodecagon; and of sixteen, a hexdecagon.

109. To find the area of a regular polygon, multiply the perpendicular falling from the centre to the middle of one of the sides by half the sum of the sides; as AB + BC + CD by mn.

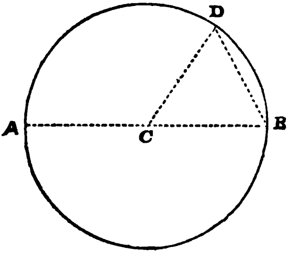


110. By having a table containing the area of regular polygons whose side is unity, the area of any similar polygon is found by multiplying the tabular area by the square of the side, similar surfaces being in proportion to the squares of their sides, as mentioned in section 90.

111. The following table exhibits the areas of regular polygons, the side being unity.

No. of Sides.	Name of Polygon.	Perpendicular or Radius of Inscribed Circle.	Radius of Circumscribing Circle.	Area.
3	Trigon or Eq. Triangle	.28867	.57735	.4330127
4	Tetragon or Square ...	.50000	.70710	1.0000000
5	Pentagon	.68819	.85065	1.7204774
6	Hexagon	.86602	1.00000	2.5980762
7	Heptagon	1.03826	1.15238	3.6339124
8	Octagon	1.20710	1.30656	4.8284272
9	Nonagon	1.37373	1.46190	6,1818242
10	Decagon..................	1.53884	1.61863	7.6942088
11	Undecagon...............	1.70284	1.77473	9.3656411
12	Dodecagon...............	1.86602	1.93185	11,1961524
16	Hexdecagon	2.51367	2.56653	20.109360

112. A circle is a plane figure bounded by a curve line called the circumference, which is everywhere equidistant from a point within it called the centre.



113. A line drawn through the centre, and meeting

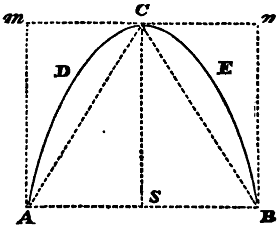
the purpose of allowing the knot to be tied at the exact place. It may be driven in at either A, B, C, or D.

121. To find the area of a circle, multiply the square of its diameter by .7854.

122. To find the area of an ellipse, multiply the transverse diameter by the conjugate, and the product by .7854.

123. The rule for each is virtually the same—that is, the product of the two diameters by .7854; for the circle bears the same ratio to the square in which it is inscribed as the ellipse bears to the rectangle in which it is inscribed, viz., as .7854 to 1.

124. To find the area of a parabola, as A D C E B, multiply the base A B by the perpendicular height C *s*, and take $\frac{2}{3}$ of the product; for a parabola is equal to $\frac{2}{3}$ of its circumscribing parallelogram.

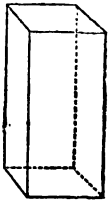


Parallelogram = 1
Parabola = $\frac{2}{3}$
Isosceles Triangle = $\frac{1}{2}$

125. As a triangle is equal to half a parallelogram,

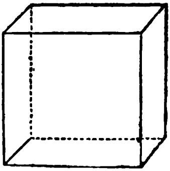
it follows that the isosceles triangle A C B is equal to $\frac{3}{4}$ of its circumscribing parabola—that is, if the area of the parallelogram *m* A B *n* be 6, that of the parabola A D C E B will be 4, and of the isosceles triangle A C B, 3.

126. A rectilineal solid whose ends are equal, similar and parallel, and whose sides are parallelograms, is called a prism.

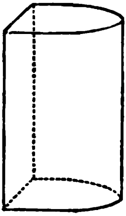


If the ends are square, it is a square prism; if triangular, a triangular prism; if a pentagon, a pentagonal prism, &c. If the ends of the prism are parallelograms, it is a parallelopiped or rectangular prism.

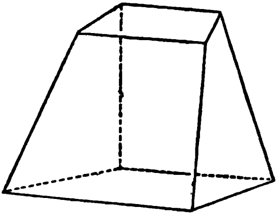
If all the sides are square, it is a cube.



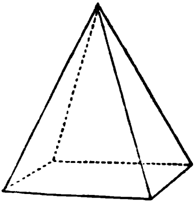
If the ends are parabolas, it is a parabolic prism.



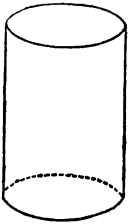
If the ends are unequal and dissimilar, it is a prismoid.



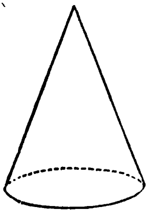
127. A pyramid is a solid which has a rectilineal base, and triangular sides meeting in a point, called the vertex.



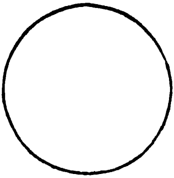
128. A cylinder is a round solid, of uniform thickness, having circular ends.



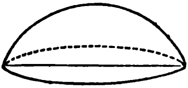
129. A cone is a round solid tapering uniformly to a point.



130. A sphere is a solid every way round.

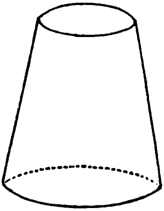


131. A segment of a solid is the part cut off the top by a plane parallel to its base.

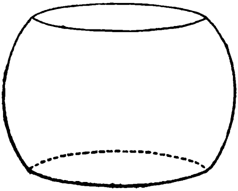


132. In the sphere any portion cut off by a plane is a segment; and the circle in which the plane cuts the sphere is the base of the segment.

133. A frustum of a solid is the part left after the segment has been cut off.



134. A zone of a sphere is that part which is intercepted between two parallel planes; and if the planes are equally distant from the centre it is called the middle zone of the sphere.



135. Prisms, cylinders, pyramids, and cones, are said to be right, or upright, when the axis is perpendicular to the base; when otherwise, they are termed oblique.

136. Plane figures formed by the cutting of a cone are called conic sections.

The sections of a cone are five in number, but three only are usually termed conic sections—the ellipse, the parabola, and the hyperbola.

If the cutting plane passes through the vertex of a right cone and any part of the base, the section is an isosceles triangle; if through the sides parallel to the base, a circle; if obliquely through the sides, an ellipse; if through one side and parallel to the other, a parabola; if in any other way, an hyperbola.

137. A cylinder cut obliquely to its axis also produces an ellipse.

138. A line perpendicular to a diameter is called an ordinate; and the sections of the diameter met by it are termed abscissas.

139. Round solids may be conceived to be generated by the rotation of planes on their diameters, as axes.

140. A rectangular parallelogram revolving on the perpendicular bisecting it produces a cylinder.

141. An isosceles triangle rotating on the perpendicular dropped from its vertex forms a cone.

142. A circle turning upon its diameter shapes out a sphere.

143. An ellipse revolving on its axis generates a spheroid, or ellipsoid.

If the ellipse turns upon its transverse diameter the spheroid is called prolate; if on the conjugate, an oblate spheroid.

144. A parabola revolving on its axis produces a paraboloid, or parabolic conoid.

145. An hyperbola turning upon its axis generates an hyperboloid, or hyperbolic conoid.

146. The revolution of the section of a curve, on a double ordinate as axis, produces a spindle, which is circular, elliptic, parabolic, or hyperbolic, according to the curve from which the section is taken.

147. Solid bodies may likewise be conceived to be generated by the motion of their bases, ascending always parallel to themselves along a right line.

148. A square thus ascending produces a square prism; a triangle, a triangular prism; a pentagon, a pentagonal prism; a parabola, a parabolic prism; a circle, a circular prism or cylinder; an ellipse, an elliptic prism or cylinder.

149. If the planes, during their ascent, diminish in the ratio of their axes till they end in a point, the generated figures will be pyramids; the square will form the square pyramid, &c.; the circle, the round pyramid or cone; the ellipse, the elliptic pyramid or cone.

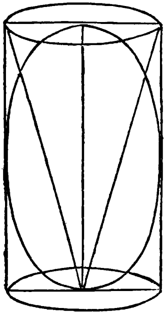
150. Spheres, spheroids, and spindles, may also be conceived to be generated by expanding and contracting circular planes, moving perpendicularly along the axis.

151. Expanding circular planes may also generate the paraboloid and hyperboloid.

152. In the following pages, however, all bodies which *can* be generated by the turning round of planes upon a central axis—all, in short, which can be formed in a lathe, will be considered as solids of rotation; and all others as solids of ascension.

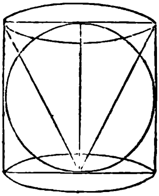
153. A solid circular at one end, and elliptical at the other, is termed a cylindroid.

154. A beautifully simple relation exists between the cone, the sphere or spheroid, and the cylinder; the altitude and diameters, or fixed and revolving

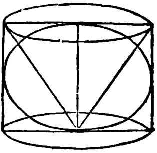


Cylinder = 1
Prolate Spheroid = $\frac{2}{3}$
Cone = $\frac{1}{3}$

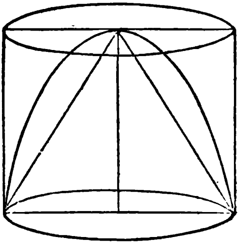
axes being the same. If the capacity of the cylinder be 1, that of the sphere or spheroid will be $\frac{2}{3}$, and of the cone $\frac{1}{3}$; or, in whole numbers, the cylinder



Cylinder	1
Sphere	$\frac{2}{3}$
Cone	$\frac{1}{3}$



Cylinder	1
Oblate Spheroid	$\frac{2}{3}$
Cone	$\frac{1}{3}$



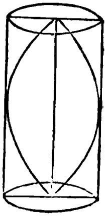
Cylinder	1
Paraboloid	$\frac{1}{2}$
Cone	$\frac{1}{3}$

being 3, the sphere or spheroid will be 2, and the cone 1.

155. An equally simple relation exists between the cylinder and the less familiar solid—the paraboloid. If the capacity of the cylinder be 1, that of the paraboloid will be $\frac{1}{2}$; or, in whole numbers, the cylinder being 2, the paraboloid will be 1.

156. The reader can compare the areas of the generating planes, the parallelogram, the parabola, and the isosceles triangle, in section 125, with the capacities of the generated solids in the present section—the cylinder, the paraboloid, and the cone. The areas were as 6, 4, 3; the solidities are as 6, 3, 2.

157. A parabolic spindle is equal to $\frac{8}{15}$ of its circumscribing cylinder.



Cylinder 1
Parabolic Spindle $\frac{8}{15}$

158. As the cone, which is a round pyramid, is $\frac{1}{3}$ of the cylinder, which is a round prism; so every pyramid is $\frac{1}{3}$ of a prism of equal base and altitude.

159. The sphere fits into a cube; the prolate spheroid into a square prism deeper than a cube; the oblate spheroid into a square prism shallower than a cube.*

160. Vessels in the form of an oblate spheroid are not met with in gauging; but the middle portions of casks assume the form of the middle frustum of the prolate spheroid, and they are always gauged as if their entire shape coincided with such figure. As the ends, however, are conical, the content so found is somewhat too great, and a compensation has to be made in deducting a portion of the length, of which hereafter.

161. As the ratio of the cylinder to the square

---

* The Earth is a spheroid slightly oblate, her polar diameter, as determined by careful measurements of a degree at different parts of her surface, being about 26 miles less than her equatorial. Jupiter is much more oblate, as readily seen through a telescope, his polar diameter being to his equatorial as 69 to 74. If it be allowable to descend from the dignity of science to illustrate by familiar examples, as is usual in Geography in comparing the inequalities of the earth's surface with the roughness of an orange, it may be stated that the damson is an approximate specimen of the prolate spheroid, and the onion of the oblate. Some eggs are almost perfect prolate spheroids, though the generality of them are too pointed at one end to allow them to come under that denomination. Lamp glasses are frequently made oblately spheroidal, a form easily imparted to them from centrifugal force, by causing them to rotate while in a semifluid condition.

prism is as their bases—that is, as the circle to the square, since the depth is the same, and only the breadth and thickness vary, consequently the ratio of the globe to its enveloping cube is $\frac{2}{3}$ of the ratio of the circle to its circumscribing square; and the ratio of the spheroid to its enveloping square prism $\frac{2}{3}$ of the ratio of the ellipse to its circumscribing parallelogram.

162. The content of prisms is found by multiplying the area of the base by the height, as before explained. Hence the content of rectangular prisms is the product of the height, breadth, and thickness.

163. The parabolic prism being $\frac{2}{3}$ of the rectangular, its capacity is obtained by multiplying the base, or double ordinate of its parabolic end, by its height, or abscissa, and by $\frac{2}{3}$ of the length of the solid.

164. The pyramid is $\frac{1}{3}$ of the prism; its solidity is therefore found by multiplying the area of the base by $\frac{1}{3}$ of the height.

165. As the area of a circle is obtained by multiplying the square of the diameter by .7854, the content of the cylinder is found by multiplying the height by the square of the diameter and by .7854.

166. The globe being $\frac{2}{3}$ of the cylinder, its capacity is found by multiplying the diameter by the square of the diameter and by $\frac{2}{3}$ of .7854—that is, the cube of the diameter by .5236; the content of the prolate spheroid by multiplying together the transverse axis,

the square of the conjugate, and .5236; and of the oblate by multiplying the conjugate by the square of the transverse, and by .5236.*

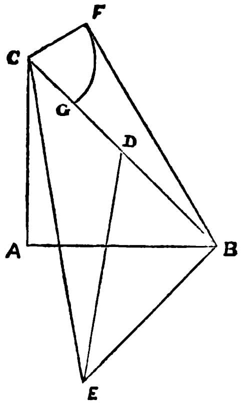
167. The cone is $\frac{1}{3}$ of the cylinder; its capacity, therefore, is obtained by multiplying the height by the square of the diameter and by $\frac{1}{3}$ of .7854—viz., .2618.

168. In like manner the content of the parabolic spindle is found by multiplying the length by the square of the diameter and by $\frac{8}{15}$ of .7854—viz., .41888.

169. So the capacity of the paraboloid is obtained by multiplying together the length, the square of the diameter, and the half of .7854—viz., 3927.

---

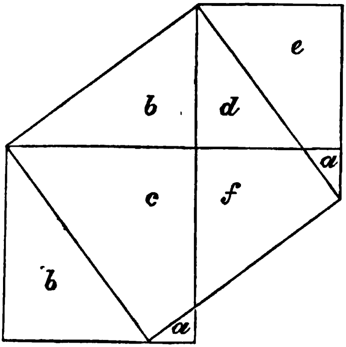
* The revolutions of a parallelogram exhibit very clearly the difference between the prolate and the oblate spheroid. Suppose a parallelogram 12 inches by 6 to be divided by two lines across the middle, at right angles to each other, so as to cut it into 4 equal portions, each 6 by 3. Then, if the parallelogram revolve on the short axis, it will generate a cylinder 6 inches deep, and having a diameter of 12 inches; consequently its content will be $6 \times 12^2 \times .7854$. If it revolve on the long axis the cylinder produced will be 12 inches deep and 6 diameter, and its content $12 \times 6^2 \times .7854$—that is, in each case, $f v^2$ .7854. Two thirds of this, or $f v^2$ .5236, gives the spheroid. If, instead of a parallelogram, a square be supposed to revolve, the depth of the cylinder will be equal to its diameter, and the circle of like diameter rotating within this produces the sphere, f becoming equal to v, and the formula passing into $d\,d^2$ .5236—viz., d^3 .5236.



170. Let the line A B be the diameter of a given cylinder.

At the point A raise the perpendicular A C, equal to A B. Join C B, and bisect it in D. At the point B draw B E, perpendicular to C B, and equal to A B. Join E D, E C. Bisect C D in G, and from C with the radius C G describe an arc at F. Draw B F a tangent to the arc, and join C F. Then, if the fixed axis of a cylinder, of a spheroid, of a parabolic spindle, of a paraboloid, and of a cone, be the same, and A B be taken as the revolving axis or diameter of the cylinder, E D the revolving axis of the spheroid, F B the revolving axis of the parabolic spindle, C B the revolving axis or diameter of the base of the paraboloid, and CE the revolving axis or diameter of the base of the cone, the solids will all be equal.

171. The proof of this depends, of course, upon the 47th of the 1st of Euclid. Of the proof of *that* the following is offered as concise. If it has appeared in the pages of any previous writer, I am not aware of it.



$$b + d = d + f \therefore b = f$$

$$\text{but} \quad b = a + e \therefore f = a + e$$

$$\therefore \underbrace{b + c + d + f}_{\textit{Square of Hypothenuse.}} = \underbrace{a + b + c}_{\textit{Square of Base.}} + \underbrace{d + e}_{\textit{Square of Perpendicular.}}$$

OF DIVISORS, FACTORS, AND GAUGE POINTS.

172. Divisors are prepared numbers, so computed as to give the required quotient by a single operation.

173. Factors are the reciprocals of divisors, employed for converting division into multiplication.

174. Gauge points are the square roots of divisors, used for the adaptation of formulæ to the D line of the Slide-rule.

175. Thus, suppose the base of a triangle to be 24 inches, and its perpendicular 18 inches, and we require its content in square feet. $24 \times 18 = 432$, the area of a parallelogram. Dividing this by 2, we have 216 square inches, the area of the triangle; and 216 divided by $144 = 1.5$ square ft. Instead, however, of dividing by 2, and then by 144, we may divide by 288 at once; and $432 \div 288 = 1.5$, as before.

Here 288 is the divisor.

176. The reciprocal of 288 is $\frac{1}{288} = .00347\dot{2}$.

Multiplying this by the length and breadth, we have

$$\begin{array}{r} .00347\dot{2} \\ 24 \\ \hline 1388\dot{8} \\ 694\dot{4}4 \\ \hline .08\dot{3}33\dot{3} \\ 18 \\ \hline 66\dot{6} \\ 8\dot{3}3 \\ \hline 1.500 \\ \hline\hline \end{array} \qquad \begin{array}{r} \text{Or thus—} \\ 24 \\ 18 \\ \hline 192 \\ 24 \\ \hline 432 \\ .00347\frac{2}{9} \\ \hline 3024 \\ 1728 \\ 1296 \\ 96 \\ \hline 1.50000 \\ \hline\hline \end{array}$$

the area as before.

Here $.00347\dot{2}$ or $.00347\frac{2}{9}$ is the factor.

177. The square root of 288 = 16.97.

In rules in which the commencement of A coincides with the commencement of D—

$$\frac{18 \times 24}{16.97^2} = ?$$

Over 16.97 on D set 18 on C, then under 24 on A is 1.5 on B;

$$\text{Or} \qquad \frac{1\sqrt{18 \times 24}^{\,2}}{16.97^2} = \frac{1 \times 20.8^2}{16.97^2} = ?$$

Set the 18 of C and the 18 of D together. Under 24 of C is 20.8, the mean proportional; then

$$16.97 \text{ D} : 1 \text{ C} :: 20.8 \text{ D} : 1.5 \text{ C}$$

Here 16.97 is the gauge point.

ON THE COMPUTATION OF DIVISORS.

178. On examining a chess-board we find it to contain 64 divisions. If each of these divisions be 1 inch square, the area of the divided part will be 64 square inches. If upon each of these divisions we place an inch cubical block of wood, we shall have a layer containing 64 cubic inches; and upon this we may pile up a number of layers, forming a square prism of any required height; the content of the whole will then be the number of blocks in each layer, multiplied by the number of layers in the pile. By taking the dimensions of perpendicular vessels, therefore, in inches, and considering the depth or thickness of each layer as 1 inch, or unity, the number of *cubic* inches in each layer will, of course, be equal to the number of *square* inches in the area of the base; s^2, (the product of the sides, representing the latter;) and $1 s^2$, (the depth multiplied by the square of the side, which denotes the former,) being identical.

179. On this account it is usual, in Gauging, to estimate the capacity of a vessel at the depth of *one inch*, and to call such *capacity* the *content of the area;*

in which expression the word area is to be considered as employed *technically*, in a sense different from its common acceptation, in which it signifies only quantity of surface, or superficial extension.

180. Having found the content of the area, it remains simply to multiply it by the number of inches in the height of the vessel to obtain the entire capacity of the vessel.

181. The first thing, therefore, to be considered, in computing the content of a vessel, is the estimation of the area of the *base*, if the vessel is *perpendicular;* or, if the sides are *protuberant* or *tapering*, such an *enlarged* or *diminished area* as would, if carried to the same height, form a vessel of equal capacity; and, on that account, termed a *mean area.*

182. To obtain the area when the base is a square, all that is required is to measure its side, and to multiply it by itself. When the base is a circle, however, the case is far different. A circle, inscribed in a square, is, of course, much less than the square itself: the square of the diameter, therefore, which is the area of the circumscribing square, will be much greater than the area of the circle. The circle, however, will bear a certain ratio to the square; and therefore we have only to ascertain this ratio to be enabled to deduce the area of the circle at once from the area of the square; and the content of the cylinder, standing upon the circular base, from the

content of the circumscribing prism placed upon the square base.

183. To obtain this ratio, the first inquiry is, what is the area of a circle whose diameter is unity? This is obtained by multiplying the semi-diameter by the semi-circumference; for the circle may be considered as a polygon with an infinite number of sides; and the area of a regular polygon is found by dividing it into as many isosceles triangles as the figure has sides. And since the area of each triangle is equal to its height multiplied by half the base, the area of the polygon will be equal to the perpendicular multiplied by the semi-perimeter; and if the number of the sides of the polygon be supposed to be indefinitely increased, its perimeter will ultimately coincide with the circumference of a circle, its perpendicular merging into the radius; and, consequently, the area will be equal to the product of the radius, or semi-diameter, by the semi-circumference.

184. The determination of the circumference from the diameter has engaged the efforts of Mathematicians for ages; to deduce it geometrically, or to exhibit it by a finite formula, has hitherto been found impracticable. It has, however, been determined with accuracy more than sufficient for all practical purposes. When the diameter is 1, the circumference is 3.1415926535; the fourth part of their product, viz., .785398, &c., or .7854, is, therefore, the area of a circle whose diameter is 1, or

the area of a circle inscribed in a square whose area is 1. Now .7854 (the area of a circle) is to 1 (the area of its circumscribing square) as 1 (the area of a second circle) is to 1.273239544 (the area of its circumscribing square); or $\frac{.7854}{1} = \frac{1}{1.273239544}$; that is, a circle inscribed in a square is the $\frac{1}{1.273239544}$th part of the square; and a cylinder, inscribed within a square prism, the $\frac{1}{1.273239544}$th part of such prism.

185. This reduces the mensuration of a solid of rotation to the simplest form possible; for we have only to imagine it fitted into a square prism of a depth equal to the fixed axis, and in such manner that the four sides shall be tangents to the revolving axis; the square of the latter multiplied by the former then gives the capacity of the prism, and this, divided by a proper divisor, the capacity of the included solid.

186. Putting, therefore, h, the height or depth of the prism $= f$, the fixed axis; and s, the side of the square base, $= v$, the revolving axis, or d, the diameter; then $h\,s^2$, or $h\,d^2$, or $f\,v^2 =$ the capacity of the square prism; and $\frac{h\,s^2}{G}$, or $\frac{h\,d^2}{G}$, or $\frac{f\,v^2}{G}$, or some modification of it, will represent the capacity of *all*

solids, G being a divisor prepared for the purpose, and varying with the nature of the solid under examination.

187. The parabolic prism, being equal to $\frac{2}{3}$ of its circumscribing rectangular prism, will be $\frac{h\,b\,a}{1} \times \frac{2}{3} = \frac{h\,b\,a}{1.5}$, where b denotes the base, or double ordinate, and a the absciss of the parabolic end.

188. A triangular prism, being $\frac{1}{2}$ its circumscribing rectangular prism, will be $\frac{h\,b\,p}{2}$, where b denotes the base of the triangular end, and p a perpendicular dropped upon it from the opposite angle.

189. The pyramid is $\frac{1}{3}$ of the prism, and is therefore denoted by $\frac{h\,s^2}{3}$

190. The cylinder, as before explained, is $\frac{h\,d^2}{1.273239544}$

191. The sphere or spheroid being $\frac{2}{3}$ of the cylinder, its capacity will be denoted by

$$\frac{d\,d^2 \text{ or } f\,v^2}{1.273239544} \times \frac{2}{3} = \frac{d\,d^2 \text{ or } f\,v^2}{1.909859316}$$

192. The cone is $\frac{1}{3}$ of the cylinder, and its capacity

$$\frac{h\,d^2}{1.273239544} \times \frac{1}{3} = \frac{h\,d^2}{3.819718632}$$

193. The paraboloid is $\frac{1}{2}$ the cylinder, and its capacity, therefore, will be

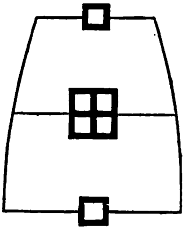
$$\frac{h\,d^2 \text{ or } f\,v^2}{1.273239544} \times \frac{1}{2} = \frac{h\,d^2 \text{ or } f\,v^2}{2.546479088}$$

194. The parabolic spindle being $\frac{8}{15}$ of its circumscribing cylinder, its content will be

$$\frac{f\,v^2}{1.273239544} \times \frac{8}{15} = \frac{f\,v^2}{2.387324145}$$

195. It now remains to speak of Frustums, and for these the following is an invaluable rule, and of general applicability.

196. Find the *area* of the top, the *area* of the bottom, and *four times* the area of a section midway between the two. Their sum is *six* times a mean area; which, being multiplied by *one-sixth* of the depth, gives the content. But considering the frustum, like the complete solid, a portion of a square prism, the rule becomes—Add together the *square* of



the top diameter, the *square* of the bottom diameter and *four times* the square of a diameter midway between the two, and multiply by *one-sixth* of the depth. But four times the square of a number is equal to the square of twice that number; therefore the rule becomes still easier: add together the square of the *top* diameter, the square of the *bottom* diameter, and the square of *twice the intermediate diameter*, and multiply by one-sixth of the depth. Moreover, when solids do not bulge in the middle, like globes and spindles, but taper regularly, like cones and pyramids, then the sum of the top and bottom *will be* twice the intermediate diameter; therefore, for all *regularly-tapering* frustums, the above given rule becomes still more concise, viz.: Add together the square of the *top* diameter, the square of the *bottom* diameter, and the square of their *sum*, and multiply by *one-sixth* of the depth. But in the same way as for cones and pyramids we multiplied by the *whole* of the height, and took *three* times the divisor, so for frustums we may take *the whole* of the height, and divide by *six* times the divisor.

197. Therefore, putting s the side of the less, and S the side of the greater square end; d, the diameter of the less, and D, the diameter of the greater circular end; then the content of the frustum of a square pyramid will be $\frac{h(s^2 + S^2 + \overline{s + S}^2)}{6}$; and of

the frustum of a cone, $\frac{h(d^2 + D^2 + \overline{d + D}|^2)}{1.273239544 \times 6} =$

$$\frac{h(d^2 + D^2 + \overline{d + D}|^2)}{7.639437264}$$

198. This is an inestimable rule, and worthy of all remembrance. It applies to every solid whose base and top are parallel, as the following examples will illustrate.

199. A carpenter has a block of wood 11 inches square at top, 13 inches square at bottom, and 12 inches deep. Does it contain an exact cubic foot, or more, or less?

$$\begin{array}{lr}
\text{Top} \quad . \quad . & 11^2 = 121 \\
\text{Bottom} \quad . & 13^2 = 169 \\
\text{Sum} \quad . \quad . & 24^2 = 576 \\
\cline{2-2}
& 866 \\
& 2 = \tfrac{1}{6} \text{ of depth.} \\
\cline{2-2}
& 1732 \\
\cline{2-2}
\end{array}$$

So it contains 4 cubic inches more than a solid foot, the excess of the four bottom corners over those of a cube being greater than the deficiency of the top corners.

200. A prismoid 24 inches deep measures 12 inches

by 10 at top, and 16 by 14 at bottom: what is the content in cubic inches?

Top . . 12 × 10 = 120
Bottom . 16 × 14 = 224
Sum . . 28 × 24 = 672
1016
4 = $\frac{1}{6}$ depth.
4064 cubic inches.

201. A prismoid 24 inches deep measures 12 inches by 10 at top, and 14 by 16 at bottom: what is the content in cubic inches?

Top . . 12 × 10 = 120
Bottom . 14 × 16 = 224
Sum . . 26 × 26 = 676
1020
4 = $\frac{1}{6}$ depth.
4080 cubic inches.

202. The dimensions of this prismoid, it will be observed, are the same as those of the preceding, but the bottom parallelogram is shifted so that the 14 comes under the 12, and the 16 under the 10; this changes the shape of the prismoid, and alters its content. In measuring prismoids, therefore, attention must be paid to the position of the sides.

203. A prismoid 24 inches deep is 14 inches square at top, and 16 by 12 at the bottom: required the content in cubic inches.

Top . . 14 × 14 = 196
Bottom . 16 × 12 = 192
Sum . . 30 × 26 = 780

1168
4 = $\frac{1}{6}$ depth.

4672 cubic inches.

204. A wedge measures 8 inches along the edge, the base is 12 inches long and 4 thick, and the perpendicular height 18 inches: what is the solidity?

Top . . 8 × 0 = 0
Bottom . 12 × 4 = 48
Sum . . 20 × 4 = 80

128
3 = $\frac{1}{6}$ height.

384 cubic inches.

205. A cylindroid, 24 inches deep, is bounded at top by a circle 6 inches in diameter, and at the bottom by an ellipse whose axes are 12 and 10 inches: required its content.

Top . . 6 × 6 = 36
Bottom . 12 × 10 = 120
Sum . . 18 × 16 = 288

444
24

1776
888

10656

$$\frac{10656}{7.639437264} = 1394.8 \text{ cubic inches.}$$

Here we have multiplied by the *whole* height, and taken the round frustum divisor, which is six times the cylindrical, as explained in section 197.

206. Required the solidity of the frustum of a hexagonal pyramid, the length of each of the sides at top being 6 inches, at the bottom 18, and the perpendicular height 40.

Top . 6 × 6 = 36
Bottom 18 × 18 = 324
Sum . 24 × 24 = 576

936
40
6|37440
6240

2.5980762
6240

1039230480
51961524
155884572
12|16211.9954880
12|1350.999624
12|112.583302
9.38 cubic feet.

Here the areas are obtained by multiplying the squares of the sides by the proper tabular number, as explained in section 111.

207. If the diameter of a globe be 1, the diameter of a midway section parallel to it, which is the chord of 120°, or twice the sine of 60°, will be $\frac{1}{2}\sqrt{3}$; twice this middle diameter will therefore be $= \sqrt{3}$: required the solidity of the globe.

$$\begin{array}{lr} \text{Square of top} & = 0 \\ \text{Square of bottom} & = 1 \\ \text{Square of twice middle} & = 3 \\ \hline & 4 \end{array}$$

And $4 \times \frac{1}{6} \times .7854 = .5236$, the content, as determined by other modes.

208. This applicability of the rule to such very dissimilar solids will serve to show its great utility. It must be carefully borne in mind, however, what is meant by intermediate diameter, for when a solid is protuberant about the centre, and runs off on either hand towards a point, by intermediate diameter is to be understood, not the diameter in the middle of the solid, but the diameter half-way between the less and greater diameter—as, for instance, in a cask, the intermediate diameter is not the bung diameter, but a diameter half-way between the bung and head; so, in a spindle, the intermediate diameters are at one-fourth of the length from each point; such figures, in reality,

consisting of two similar solids joined together at their bases, and therefore having two intermediate diameters.

209. As the previous formulæ give the content in the denomination in which the dimensions are taken, the divisors will require a modification if the capacity is sought in a different name. For instance, if the dimensions are taken in inches, and the content is required in cubic feet, then the divisors must be multiplied by 1728, that being the number of cubic inches in a cubic foot. The formula for the square prism will accordingly be $\frac{h\,s^2}{1728}$, and for the cylinder $\frac{h\,d^2}{1.273239 \times 1728} = \frac{h\,d^2}{2200.16}$. Supposing the content required in gallons, then, since a gallon contains 277.274 cubic inches, we have for the square prism $\frac{h\,s^2}{277.274}$; and for the cylinder, $\frac{h\,d^2}{1.273239 \times 277.274}$ $= \frac{h\,d^2}{353.036}$

A bushel contains 8 gallons; consequently, multiplying these by 8, we have the divisors for bushels. Having found the divisors, their reciprocals furnish the factors, and their square roots the gauge points. Divisors for polygonal prisms are found by dividing 277.274 by the polygonal numbers in section 111; the factors, by dividing those numbers by 277.274.

210. TABLE OF DIVISORS, FACTORS, AND GAUGE POINTS.

	SOLIDS OF ASCENSION, TRILATERAL AND QUADRILATERAL.					SOLIDS OF ROTATION, ROUND OR INNUMERILATERAL.					
RATIOS	1.					.7854					
Dimensions to be taken in Inches.	Rectangle, Square, or Square Prism.	Parabola or Parabolic Prism.	Triangle or Triangular Prism.	Rect-angular or Square Pyramid.	Square Frustum, or Prismoid.	Circle, Ellipse, or Cylinder.	Sphere or Spheroid.	Parabolic Spindle.	Paraboloid or Parabolic Conoid.	Circular or Elliptic Cone.	Round Frustum.
SUBRATIOS	1	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$	1	$\frac{2}{3}$	$\frac{8}{15}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$
GALLONS.											
Divisors	277.274	415.91	554.548	831.822	1663.644	353.036	529.554	661.94	706.072	1059.108	2118.216
Factors	.0036065	.002404	.00180325	.0012022	.0006011	.00283257	.0018884	.0015107	.0014162	.0009442	.0004721
Gauge Points	16.65^2	20.39^2	23.54^2	28.84^2	40.79^2	18.79^2	23^2	25.72^2	26.57^2	32.54^2	46^2
BUSHELS.											
Divisors	2218.192	3327.28	4436.385	6654.576	13309.15	2824.29	4236.435	5295.54	5648.58	8472.87	16945.74
Factors	.00045081	.00030054	.0002254	.00015027	.00007513	.00035407	.00023605	.00018884	.00017703	.00011802	.00005901
Gauge Points	47.1^2	57.68^2	66.6^2	81.57^2	115.36^2	53.14^2	65.08^2	72.77^2	75.15^2	92^2	130.17^2
CUBIC FEET.											
Divisors	1728	2592	3456	5184	10368	2200.16	3300.24	4125.3	4400.32	6600.48	13200.96
Factors	.0005787	.0003858	.00028935	.0001929	.0000965	.000454	.000302	.0002421	.000227	.000151	.000076
Gauge Points	41.56^2	50.9^2	58.78^2	72^2	101.8^2	46.9^2	57.45^2	64.22^2	66.3^2	81.24^2	114.9^2

211. TABLE OF DIVISORS, FACTORS, & GAUGE POINTS FOR REGULAR POLYGONS.

Dimensions to be taken in Inches, Content in Gallons.	SOLIDS OF ASCENSION, MULTILATERAL.		
	Prism.	Pyramid.	Frustum.
Trigonal	640.34 .001562 25.3^2	1921.01 .000521 43.8^2	3842.02 .000261 61.97^2
Tetragonal	277.274 .003607 16.65^2	831.82 .001202 28.84^2	1663.64 .000601 40.79^2
Pentagonal	161.16 .006205 12.7^2	483.48 .002068 21.98^2	966.96 .001034 31.1^2
Hexagonal	106.72 .009370 10.33^2	320.16 .003123 17.9^2	640.32 .001562 25.33^2
Heptagonal	76.39 .013106 8.73^2	229.17 .004369 15.13^2	458.34 .002184 21.4^2
Octagonal	57.42 .017414 7.57^2	172.27 .005805 13.12^2	344.54 .002902 18.55^2
Nonagonal	44.85 .022295 6.7^2	134.56 .007432 11.6^2	269.12 .003716 16.4^2
Decagonal	36.03 .027740 $6.^2$	108.11 .009247 10.39^2	216.22 .004624 14.7^2
Undecagonal	29.6 .033778 5.44^2	88.81 .011259 9.42^2	177.63 .005630 13.33^2
Dodecagonal	24.76 .040379 4.97^2	74.3 .013460 8.6^2	148.6 .006730 12.12^2

PRACTICAL GAUGING.

MENSURATION OF AREAS.

Problem 1.

212. To find the area of a square.

Rule.—Multiply the side by itself, and take the divisors, factors, and gauge points, standing in the table under the word square.

FORMULÆ.

By divisors—

$$\frac{s^2}{277.274} = \text{gallons}, \qquad \frac{s^2}{2218.192} = \text{bushels}.$$

By factors—

$$.0036065\ s^2 = \text{gallons}, \qquad .00045081\ s^2 = \text{bushels}.$$

By gauge points—

$$\frac{1\ s^2}{16.65^2} = \text{gallons}, \qquad \frac{1\ s^2}{47.1^2} = \text{bushels}.$$

EXAMPLES.

213. Required the number of gallons contained in a vessel 60 inches square, 1 inch deep.

$$\frac{60 \times 60}{277.274} = 12.98 \text{ gallons},$$

$$.0036065 \times 60 \times 60 = 12.98 \text{ gallons}.$$

By the slide-rule—

$$\frac{1 \times 60^2}{16.65^2} = ?$$

16.65 D : 1 C :: 60 D : 12.98 C

214. Required the number of bushels in a vessel 90 inches square, 1 inch deep.

$$\frac{90 \times 90}{2218.192} = 3.65 \text{ bushels},$$

$$.00045081 \times 90 \times 90 = 3.65 \text{ bushels}.$$

$$\frac{1 \times 90^2}{47.1^2} = ?$$

47.1 D : 1 C :: 90 D : 3.65 C

215. A vessel is 105 inches square: required its area in bushels.—*Ans.* 4.97.

216. A vessel is 40 inches square: required its area in gallons.—*Ans.* 5.77.

Problem 2.

217. To find the area of a parallelogram.

Rule.—Multiply the length by the perpendicular breadth, and take the divisors, factors, and gauge points, under the word square or rectangle.

Formulæ.

By divisors—

$$\frac{lb}{277.274} = \text{gallons}, \qquad \frac{lb}{2218.192} = \text{bushels}.$$

By factors—

$.0036065\,l\,b =$ gallons, $.00045081\,l\,b =$ bushels.

By gauge points—

$$\frac{1\ q^2}{16.65^2} = \text{gallons}, \qquad \frac{1\ q^2}{47.1^2} = \text{bushels},$$

where l denotes length, b breadth, and q the square root of their product, or mean proportional between them.

EXAMPLES.

218. The length of a parallelogram is 50 inches, its breadth 30: required its area in gallons.

$$\frac{50 \times 30}{277.274} = 5.4 \text{ gallons},$$

$$.0036065 \times 50 \times 30 = 5.4 \text{ gallons}.$$

By the slide-rule—

$$\frac{1\sqrt{50 \times 30}^2}{16.65^2} = \frac{1 \times 38.73^2}{16.65^2} = ?$$

Set 50 of C and 50 of D together; under 30 of C is 38.73, the mean proportional.

Then 16.65 D : 1 C :: 38.73 D : 5.4 C.

219. The side of a rhombus is 40 inches, its perpendicular breadth 37 inches: required its area in gallons.—*Ans*. 5.33.

220. The length of a parallelogram is 80 inches,

its breadth 64: required its area in gallons.—*Ans.* 18.4.

221. The length of a rhomboid is 60 inches, its perpendicular breadth 37: required its area in bushels.—*Ans.* 1.

Problem 3.

222. To find the area of a triangle.

RULE.—Multiply the base by the perpendicular breadth, and take the divisors, factors, and gauge points, standing under the word triangle.

FORMULÆ.

By divisors—

$$\frac{b\,p}{554.548} = \text{gallons}, \qquad \frac{b\,p}{4436.384} = \text{bushels}.$$

By factors—

$$.00180325\, b\,p = \text{gallons}, \qquad .0002254\, b\,p = \text{bushels}.$$

By gauge points—

$$\frac{1\,q^2}{23.54^2} = \text{gallons}, \qquad \frac{1\,q^2}{66.6^2} = \text{bushels}.$$

EXAMPLES.

223. The base of a triangle is 60 inches, its perpendicular breadth 23.5: required its area in gallons.

$$\frac{60 \times 23.5}{554.548} = 2.54 \text{ gallons},$$

$$.00180325 \times 60 \times 23.5 = 2.54 \text{ gallons}.$$

By the slide-rule—

$$\frac{1\sqrt{60 \times 23.5}^2}{23.54^2} = \frac{1 \times 37.55^2}{23.54^2} = ?$$

Set the 60 of C and the 60 of D together; under 23.5 of C is 37.55, the mean proportional.

Then 23.54 D : 1 C :: 37,55 D : 2.54 C.

224. A triangle measures 100 inches along its base, and its perpendicular is 60 inches: required its area in bushels.—*Ans.* 1.35.

225. Required the area, in gallons, of a triangle whose base is 88 inches, and perpendicular 49.—*Ans.* 7.78.

Problem 4.

226. To find the area of a trapezium.

Rule.—Divide it into two triangles, and let fall perpendiculars upon the diagonal. Multiply the sum of the perpendiculars by the diagonal, and employ the divisors, factors, and gauge points, under the word triangle.

FORMULÆ.

By divisors—

$$\frac{(p + P)d}{554.548} = \text{gallons}, \qquad \frac{(p + P)d}{4436.384} = \text{bushels},$$

By factors—

$$.00180325\ (p + P)d = \text{gallons},$$

$$.0002254\ (p + P)d = \text{bushels}.$$

By gauge points—

$$\frac{1\ q^2}{23.54^2} = \text{gallons}, \qquad \frac{1\ q^2}{66.6^2} = \text{bushels},$$

where p and P denote the perpendiculars, d the diagonal, and q a mean proportional between the diagonal and the sum of the perpendiculars.

EXAMPLES.

227. The diagonal of a trapezium is 100 inches, and the perpendiculars falling upon it 60 and 50 inches: required its area in bushels.

$$\frac{(60 + 50)100}{4436.384} = \frac{110 \times 100}{4436.384} = 2.47 \text{ bushels},$$

$$.0002254 \times 110 \times 100 = 2.47 \text{ bushels}.$$

By the slide-rule—

$$\frac{1\overline{\sqrt{110 \times 100}}|^2}{66.6^2} = \frac{1 \times 104.8^2}{66.6^2} = ?$$

Set the 110 of C and the 110 of D together; under 100 of C is 104.8, the mean proportional.

Then 66.6 D : 1 C :: 104.8 D : 2.47 C.

228. The diagonal of a trapezium is 60 inches; the perpendiculars falling upon it are 27 and 15 inches: required its area in gallons.—*Ans.* 4.54.

EXAMPLES.

234. The side of a regular hexagon is 50 inches: required its area in gallons.

$$\frac{50^2}{106.72} = 23.425 \text{ gallons.}$$

$$.00937 \times 50^2 = 23.425$$

By the slide-rule—

$$\frac{1 \times 50^2}{10.33^2} = ?$$

$$10.33 \text{ D} : 1 \text{ C} :: 50 \text{ D} : 23.42 \text{ C}$$

235. The side of a regular octagon is 47 inches: required its area.

$$\frac{47^2}{57.42} = 38.47 \text{ gallons.}$$

$$.017414 \times 47^2 = 38.47$$

By the slide-rule—

$$\frac{1 \times 47^2}{7.57^2} = ?$$

$$7.57 \text{ D} : 1 \text{ C} :: 47 \text{ D} : 38.5 \text{ C}$$

236. Required the content of the area of a regular hexagon, the side of which is 46 inches.—*Ans.* 19.826.

Problem 7.

237. To find the area of a circle.

RULE.—Square the diameter, and take the divisors, factors, and gauge points, under the word circle.

FORMULÆ.

By divisors—

$$\frac{d^2}{353.036} = \text{gallons}, \qquad \frac{d^2}{2824.29} = \text{bushels}.$$

By factors—

$$.00283257\ d^2 = \text{gallons}, \qquad .00035407\ d^2 = \text{bushels}.$$

By gauge points—

$$\frac{1\ d^2}{18.78^2} = \text{gallons}, \qquad \frac{1\ d^2}{53.14^2} = \text{bushels}.$$

EXAMPLES.

238. The diameter of a circle is 45 inches, and a rim round it is 1 inch deep: required its content in gallons.

$$\frac{45 \times 45}{353.036} = 5.73 \text{ gallons},$$

$$.00283257 \times 45 \times 45 = 5.73 \text{ gallons}.$$

By the slide-rule—

$$\frac{1 \times 45^2}{18.78^2} = ?$$

18.78 D : 1 C :: 45 D : 5.73 C

239. The diameter of a circle is 100 inches: required its area in bushels.—*Ans.* 3.54.

240. The diameter of a circle is 84 inches: required its area in gallons.—*Ans.* 20.

Problem 8.

241. To find the area of the sector of a circle.

Rule.—Multiply the length of the arc by the radius, and use the divisors, factors, and gauge points, for triangles.

242. The length of an arc of a circle may be found as follows; but where the tape can be used it is better to measure it.

243. From 8 times the chord of $\frac{1}{2}$ the arc subtract the chord of the whole arc; $\frac{1}{3}$ of the remainder is the length of the arc, nearly.

Formulæ.

By divisors—

$$\frac{r\,l}{554.548} = \text{gallons}, \qquad \frac{r\,l}{4436.385} = \text{bushels}.$$

By factors—

$$.00180325\ r\,l = \text{gallons}, \qquad .0002254\ r\,l = \text{bushels}.$$

By gauge points—

$$\frac{1\ q^2}{23.54^2} = \text{gallons}, \qquad \frac{1\ q^2}{66.6^2} = \text{bushels}.$$

Examples.

244. The radius of a circle is 40 inches; the length of the arc 32 inches: required the area of the sector in gallons.

$$\frac{40 \times 32}{554.548} = 2.3 \text{ gallons},$$

$$.00180325 \times 40 \times 32 = 2.3 \text{ gallons}.$$

By the slide-rule—

$$\frac{1\sqrt{40 \times 32}^{\,2}}{23.54^2} = \frac{1 \times 35.77^2}{23.54^2} = ?$$

Set the 40 of C and the 40 of D together; under 32 of C is 35.77, the mean proportional.

Then 23.54 D : 1 C :: 35.77 D : 2.3 C.

245. The radius of a circle is 63 inches, the length of the arc 40 inches: required the area of the sector in gallons.—*Ans.* 4.544.

Problems relating to the Circle.

246. The diameter multiplied by 3.1416 gives the circumference.

247. The diameter multiplied by .8862 gives the side of an equal square.

248. The diameter multiplied by .7071 gives the side of an inscribed square.

249. The circumference multiplied by .3183 gives the diameter.

250. The circumference multiplied by .2821 gives the side of an equal square.

251. The circumference multiplied by .2251 gives the side of an inscribed square.

252. The side of a square multiplied by 1.128 gives the diameter of a circle of equal area.

253. The side of a square multiplied by 3.545 gives the circumference of a circle of equal area.

254. The area of a circle multiplied by .6366 gives the area of an inscribed square; the square root of this is, of course, the side of the inscribed square.

255. The area of a square multiplied by .7854 gives the area of an inscribed circle.

256. The diameter multiplied by one-fourth of the circumference gives the area of the circle.

257. The square of the diameter multiplied by .7854 gives the area.

258. The square of the circumference multiplied by .07958 gives the area.

259. The circumference multiplied by .318309 gives the diameter.

260. The proportion of the diameter to the circumference is nearly as 7 to 22.

261. The proportion of the diameter to the circumference is still more nearly shown by setting down the first three odd units in pairs, and taking the first three figures for the diameter, and the last three for the circumference—viz., 113 : 355. This ratio is easily recollected, and is remarkably correct, being true to six places of decimals.

Problem 9.

262. To find the area of an ellipse.

Rule.—Multiply the transverse diameter by the conjugate, and use the divisors, factors, and gauge points, under circle or ellipse.

FORMULÆ.

By divisors—

$$\frac{t\,c}{353.036} = \text{gallons}, \qquad \frac{t\,c}{2824.29} = \text{bushels}.$$

By factors—

$$.00283257\,t\,c = \text{gallons}, \qquad .00035407\,t\,c = \text{bushels}.$$

By gauge points—

$$\frac{1\,q^2}{18.78^2} = \text{gallons}, \qquad \frac{1\,q^2}{53.14^2} = \text{bushels}.$$

EXAMPLES.

263. The transverse diameter of an ellipse is 72 inches, its conjugate 50: required its area in gallons.

$$\frac{72 \times 50}{353.036} = 10.2 \text{ gallons},$$

$$.00283257 \times 72 \times 50 = 10.2 \text{ gallons}.$$

By the slide-rule—

$$\frac{1\sqrt{72 \times 50}^{\,2}}{18.78^2} = \frac{1 \times 60^2}{18.78^2} = ?$$

Set the 72 of C and the 72 of D together; under 50 of C is 60, the mean proportional.

Then 18.78 D : 1 C : 60 D : 10.2 C.

264. The transverse diameter of an ellipse is 24

inches, its conjugate 18: required its area in gallons. —*Ans.* 1.22.

265. The transverse diameter of an ellipse is 85.9 inches, and the conjugate 63.8 inches: required its area in bushels.—*Ans.* 1.94.

PROPERTIES OF THE ELLIPSE.

266. The square of the eccentricity is equal to the difference of the squares of the semi-diameters, or to one-fourth the difference of the squares of the diameters—that is,

$$e^2 = \left.\overline{\frac{t}{2}}\right|^2 - \left.\overline{\frac{c}{2}}\right|^2 = \frac{t^2 - c^2}{4}$$

267. Consequently, the eccentricity is equal to half the square root of the product of the sum and difference of the diameters—that is,

$$e = \tfrac{1}{2}\sqrt{t^2 - c^2} = \tfrac{1}{2}\sqrt{(t + c)(t - c)}$$

268. The sum of any two lines drawn from the foci to the curve is equal to the transverse diameter—that is,

$$l + \mathrm{L} = t$$

269. Hence the distance of either focus from the extremity of the conjugate, or minor axis, is equal to the semi-major; for if $l = \mathrm{L}$, then $t = 2\,l$.

$$\therefore l = \tfrac{1}{2}\,t$$

270. The parameter, or double ordinate that passes

through the focus, is equal to the square of the conjugate divided by the transverse—that is,

$$p = c^2 \div t$$

271. The rectangles of the abscissas, or sections of any diameter, are as the squares of their ordinates—that is,

$$a\,A \propto o^2$$

272. All the parallelograms circumscribed about an ellipse are equal to one another; consequently, equal to the rectangle of the axes—that is, every circumscribing parallelogram $= t\,c$.

273. Hence ellipses are to one another as the rectangles of their axes—that is,

$$t\,c \propto T\,C$$

274. Hence, also, an ellipse is to its circumscribing parallelogram as a circle to its circumscribing square—that is,

$$t\,c : T\,C :: d^2 : D^2$$

275. The major axis of an ellipse is to the minor as the circle upon the major is to the ellipse itself; and the minor to the major as the circle upon the minor to the ellipse itself—that is,

$$t : c :: t^2 : t\,c$$

$$\text{And} \quad c : t :: c^2 : t\,c$$

276. Hence the ellipse is a mean proportional between the circles upon the two axes—that is,

$$c^2 : t\,c :: t\,c : t^2$$

Problem 10.

277. To find the area of a parabola.

RULE.—Multiply the base by the height, and use the divisors, factors, and gauge points, standing under the word parabola.

FORMULÆ.

By divisors—

$$\frac{b\,h}{415.91} = \text{gallons}, \qquad \frac{b\,h}{3327.28} = \text{bushels}.$$

By factors—

$$.002404\,b\,h = \text{gallons}, \qquad .00030054\,b\,h = \text{bushels}.$$

By gauge points—

$$\frac{1\ q^2}{20.39^2} = \text{gallons}, \qquad \frac{1\ q^2}{57.68^2} = \text{bushels}.$$

EXAMPLES.

278. The base or double ordinate of a parabola is 60 inches, its height or abscissa 45: required its area in gallons.

$$\frac{60 \times 45}{415.91} = 6.49 \text{ gallons},$$

$$.00030054 \times 60 \times 45 = 6.49 \text{ gallons}.$$

By the slide-rule—

$$\frac{1\,\overline{\sqrt{60 \times 45}}\,^2}{20.39^2} = \frac{1 \times 51.96^2}{20.39^2} = ?$$

Set the 60 of C and the 60 of D together; under 45 of C is 51.96, the mean proportional.

Then 20.39 D : 1 C :: 51.96 D : 6.49 C.

279. The base of a parabola is 120 inches, its height 96: required its area in bushels.—*Ans.* 3.46.

PROPERTIES OF THE PARABOLA.

280. The distance of the focus from the vertex is equal to the square of any ordinate divided by four times its absciss—that is,

$$d = \frac{o^2}{4\,a}$$

281. The parameter, or double ordinate at the focus, is equal to the square of any ordinate divided by its absciss—that is,

$$p = \frac{o^2}{a}$$

282. Consequently, the distance of the focus from the vertex is equal to one-fourth of the parameter—that is,

$$d = \frac{p}{4}$$

283. The parameter is to the sum of any two ordinates as the difference of those ordinates is to the difference of their abscissas—that is,

$$p : \mathrm{O} + o :: \mathrm{O} - o : \mathrm{A} - a$$

284. Abscissas are to each other as the squares of their ordinates—that is,

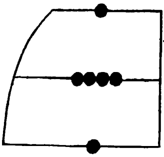
$$\mathrm{A} \propto \mathrm{O}^2.$$

285. Hence the squares of equidistant ordinates are in Arithmetical Progression.

Problem 11.

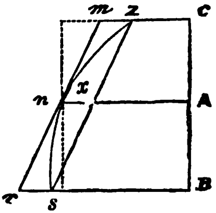
286. To find the area of a quadrilateral contained between two parallel straight lines, and bounded on one side by a parabolic curve, on the other by a straight line at right angles to the parallels.

Rule.—Add together the *breadth* of the less end, the *breadth* of the greater, and *four times* the intermediate breadth; their sum is *six* times a mean breadth, which, multiplied by *one-sixth* of the length, gives the area.



Demonstration.

287. Let $z\,n\,s$ be any portion of a parabola, cut off by the right line $z\,s$. Bisect $z\,s$ in x, and draw $n\,x$ parallel to the axis of the curve; then $n\,x$ will be the



diameter to the point n, and $z\,s$ will be an ordinate to that diameter. Through m draw $m\,r$, parallel to $z\,s$, and it will be a tangent to the point n. Draw $m\,z$ and $r\,s$ parallel to $n\,x$, and prolong the three to meet the straight line C A B, at right angles.

By reason of the parallels, $\mathrm{C\,A} = \mathrm{A\,B}$.
Now $2\,n\,\mathrm{A}.\mathrm{A\,B} =$ the trapezoid $m\,r\,\mathrm{B\,C}$.
$\overline{z\,\mathrm{C} + s\,\mathrm{B}}.\mathrm{A\,B} =$ the trapezoid $z\,s\,\mathrm{B\,C}$.
From the former subtract the latter; and
$\overline{2\,n\,\mathrm{A} - z\,\mathrm{C} - s\,\mathrm{B}}.\mathrm{A\,B} =$ the parallelogram $m\,r\,s\,z$.
$\frac{2}{3}$ of this gives the parabolic space $z\,n\,s\,z$—viz.,

$$\overline{4\,n\,\mathrm{A} - 2\,z\,\mathrm{C} - 2\,s\,\mathrm{B}}.\frac{\mathrm{A\,B}}{3}$$

To this add the trapezoid $z\,s\,\mathrm{B\,C}$—viz., $\overline{z\,\mathrm{C} + s\,\mathrm{B}}.\mathrm{AB}$;

Or, which is the same, $\overline{3\,z\,\mathrm{C} + 3\,s\,\mathrm{B}}.\frac{\mathrm{A\,B}}{3}$; and we have

$$\overline{z\,\mathrm{C} + 4\,n\,\mathrm{A} + s\,\mathrm{B}}.\frac{\mathrm{A\,B}}{3}, \text{ or } \overline{z\,\mathrm{C} + 4\,n\,\mathrm{A} + s\,\mathrm{B}}.\frac{\mathrm{C\,B}}{6};$$

that is, the sum of the less breadth, the greater breadth, and four times the intermediate breadth, multiplied by one-sixth of the length; or, taking the ordinates or breadths in the order of the letters of the alphabet, and putting A = area, and L = length,

$$\mathrm{A} = \frac{\mathrm{L}(1\,a + 4\,b + 1\,c)}{1 + 4 + 1};$$

the denominator being equal to the sum of the coefficients of the numerator, a peculiarity necessarily pervading all the formulæ for incomplete figures, whether planes or solids.

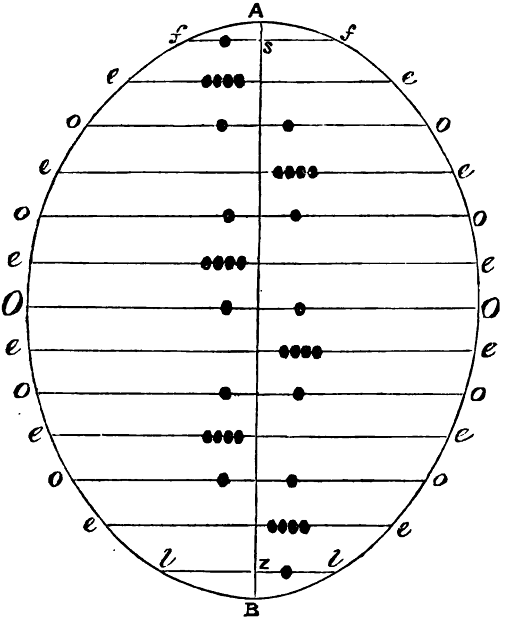
Problem 12.

288. To ascertain the area of an irregularly curved figure by an odd number of equidistant ordinates.

As oval vessels are seldom, if ever, truly elliptical, it is desirable to have some general mode of measuring all figures bounded by irregular curves; and the method of approximating areas by means of equidistant ordinates, just explained, has been adopted by the Board of Inland Revenue, and published under their authority. It appears first to have been presented, in a practical form, by Thomas Simpson, in his Mathematical Dissertations, in 1743, in which he states that "the method was originally an invention of Sir Isaac Newton's, since prosecuted by De Moivre, Stirling, and others; and that he has put the matter in such a light as he judged would be most plain and satisfactory to the reader." It is believed, however, notwithstanding this candid disavowal of originality, that the modification of Newton's plan, in which the distinctive character and real value of the rule consists, is Simpson's own.

289. When the boundary of the figure is a parabola, or a straight line, it gives the correct area. As

applied to other figures, it proceeds upon the assumption, that, as cones may be formed with every possible variety of vertical angle, no curve *can* be produced, which will not, *for a short distance*, coincide with the curve of a parabola, a figure that, from the relation contained between it and its circumscribing parallelogram, affords greater facilities for mensuration than any other of the conic sections.



290. But as a curve, which is *not* parabolic, will

more or less diverge from it, it is obvious, that the nearer the ordinates are taken to each other, the more correctly will the area be obtained.

291. Taking, therefore, a surface in detail, and considering it as made up of successive subdivisions, consisting each of two spaces between three equidistant parallel straight lines, and bounded on each side by a parabolic curve (for the figure explained in Problem 11 may be conceived to be turned over so that the right becomes a duplicate of the left) we at once obtain the area by adding together the *first* parallel, the *third* parallel, and *four* times the second or intermediate parallel, and multiplying by *one-sixth* of the distance between the extreme parallels—that is, by *one-third* of the distance between parallel and parallel. Then, proceeding with the second pair of spaces, to which the third parallel now becomes the first boundary, we have for its area the sum of the *third* parallel, the *fifth* parallel, and *four* times the fourth parallel, multiplied by *one-third* of the distance between the parallels; and thus continuing, the area of the whole figure, $f\,O\,l\,z\,l\,O\,f\,s\,f$, between the first and last parallels is eventually obtained. And here it will be perceived, that, as the spaces are taken in pairs, there will always be an *even* number of them, contained, of course, between an *odd* number of parallels. It will likewise be evident that every *odd* parallel, except the *first* and *last*, will be taken over

twice, it being a boundary common to the space *preceding* and the space *following;* and so they will run on,

$$\frac{L(1\,a + 4\,b + 1\,c)}{1 + 4 + 1};\ \frac{L(1\,a + 4\,b + 2\,c + 4\,d + 1\,e)}{1 + 4 + 2 + 4 + 1};$$

$$\frac{L(1\,a + 4\,b + 2\,c + 4\,d + 2\,e + 4\,f + 1\,g)}{1 + 4 + 2 + 4 + 2 + 4 + 1};$$

1, 4, 2, 4, 2, 4, 2, 4, 2, 4, 2, 4, 1, &c.

292. The oval, however, is not yet completed, there being a small space contained between the first and last ordinates, *f f*, *l l*, and the curves A and B. These curves are considered also as made up of parabolas.

293. Now, if we multiply the sum of the ordinates by one of the abscissas, we shall have the area of the two circumscribing parallelograms; consequently, if we multiply their sum by the sum of *both* the heights or abscissas, we shall have *twice* the area of the circumscribing parallelograms; one-third of this, of course, gives the area of the two parabolas.

294. Hence the rule for obtaining the area of an irregularly-curved surface is as follows:—

Place in a line the figures $\underline{4}$, $\underline{2}$, $\underline{1}$, and draw a mark under each.

Under the 4 place all the *even* parallels; under the 2 all the *odd* parallels, except the first and last; and under the 1 the first and last parallels.

Add up the three columns separately, and multiply each column by the figure standing over it. Add up these three products, and multiply the sum by the common distance between the parallels, or double ordinates. This is for the interordinate spaces bounded by the first and last ordinates, *f f* and *l l*, and the curves on either hand, *f* O *l*, *f* O *l*. Then for the end areas, or exordinate spaces, *f s f* A, *l z l* B, multiply the sum of the first and last ordinates standing under the figure 1 by the sum of the two abscissas. Add this product to the other, divide by the square pyramid divisor, or multiply by its factor, and the area is obtained.

295. The reason for choosing the square pyramid divisors and factors is obvious, as we have multiplied by the *whole* of the common distance, instead of by *one-third* of it.

296. It may, perhaps, occur to the reader, that we ought to have chosen the conical divisor, as being one-third of the elliptical, since the figure is of that shape; but it must be recollected that the elliptical divisor is calculated on the assumption that the transverse diameter is to be multiplied by the conjugate, which product is the area, not of the ellipse, but of its circumscribing parallelogram; and that the obtaining from that area the area of the ellipse requires the additional operation of using the augmented divisor; but by the rule just explained, for the irregular oval, we obtain thrice the area at once, without reference

to any circumscribing parallelogram of which it may be considered a portion; hence the rectangular pyramid divisor or factor is used, since a pyramid is equal to a prism of like height standing on a third of its base; and the area of a rectangle is obtained by multiplying the length by the breadth, as we have here multiplied the sum of the mean lengths of the parallels by the breadth or common distance between them.

297. As 180° are the limit of the vertical angle of a cone, a straight line is the limit of the parabolic curve; hence the rule just given will apply when the figure is a trapezoid, as is evident also by an inspection of the diagram in problem 11; for, as the curve, $z\,n\,s$, approaches to a straight line, the line, $m\,r$, approaches to the line, $z\,s$, and when the curve passes into a straight line, the line, $m\,r$, becomes coincident with the line, $z\,s$, and $n\,\mathrm{A}$ becomes equal to $x\,\mathrm{A}$;

$$\therefore \overline{z\mathrm{C} + 4n\mathrm{A} + s\mathrm{B}}.\frac{\mathrm{CB}}{6} = \overline{z\mathrm{C} + 4x\mathrm{A} + s\mathrm{B}}.\frac{\mathrm{CB}}{6} =$$

$$\overline{z\mathrm{C} + 2\,z\mathrm{C} + 2\,s\mathrm{B} + s\,\mathrm{B}}.\frac{\mathrm{CB}}{6} = \overline{z\,\mathrm{C} + s\,\mathrm{B}}.\frac{\mathrm{CB}}{2},$$

which latter is the usual formula for a trapezoid, and much simpler; so that, though the rule will apply to trapezoids and triangles, it is by no means to be adopted; for, as in these figures, the breadths are in arithmetical progression, and twice the middle is equal to the sum of the extremes, and, of course, four times the middle to twice the sum of the extremes, the adoption of the above rule for figures wholly rectilineal would be simply adding the sum of the extremes to twice the sum of the extremes, for the purpose of having the trouble to divide by 3 to obtain the sum of the extremes back again.

EXAMPLES OF AREAS BY EQUIDISTANT ORDINATES.

298. Required the area of a curvilineal figure of which the ordinates, measured in succession, are 11.2 ; 24 ; 30.735 ; 35.093 ; 37.888 ; 39.485 ; 40 ; 39.485 ; 37.888 ; 35.093 ; 30.735 ; 24 ; and 11.2 inches; the common distance between the ordinates 4 inches; and the abscissa, or height, of each terminal segment, 1 inch.

ORDINATES.					
11.2	′	4		2	1
24.	′′′′	24.		30.735	11.2
30.735	′′	35.093		37.888	11.2
35.093	′′′′	39.485		40.	22.4
37.888	′′	39.485		37.888	2
39.485	′′′′	35.093		30.735	44.8
40.	′′	24.		177.246	
39.485	′′′′	197.156		354.492	
37.888	′′	788.624			
35.093	′′′′	354.492			
30.735	′′	22.4			
24.	′′′′	1165.516			
11.2	′	4			
		4662.064	thrice the interordinate areas.		
		44.8	thrice the exordinate areas.		
		4706.864	thrice the total area.		

4706.864
.0012022 square pyramid gallon factor.

9413728
9413728
56482368

5.6585919008 content in gallons.

299. This is a true ellipse, the ordinates of which have been obtained by computation, for the purpose of proving the correctness of the rule.

300. As there are 12 common distances of 4 inches each, and 1 inch at each end for the height of the abscissas, the transverse diameter is 50 inches; the conjugate is 40, as shown by the greatest ordinate: required its area.

By the rule given in Problem 9—

.00283257
2000 product of diameters.

5.66514000 true content.
5.65859

.00655 difference, or about the $\frac{1}{153}$rd part of a gallon, or $1\frac{4}{5}$ cubic inches.

301. In the circle the rectangles of the segments of any diameter are *equal* to the squares of their ordinates; in the ellipse the rectangles of the segments of any diameter are *proportional* to the

squares of their ordinates. Hence, the ordinates of both are shown on the D line of the Slide-rule.

302. For the circle, lay the slide evenly in; then under the rectangle of any two abscissas, taken on C, is the corresponding ordinate on D. Or, if the 1 of C be set over the 2 of D, then under the rectangle of any two abscissas on C is the double ordinate on D.

303. For the ellipse, set the square of the semi-transverse on C over the semi-conjugate on D; then under the rectangle of any two abscissas on C is the corresponding ordinate on D. Or, if the square of the semi-transverse on C be set over the conjugate on D; then under the rectangle of any two abscissas on C is the double ordinate on D.

304. In the ellipse whose area we have just calculated, the first double ordinate crosses the transverse 49 inches from one end, and 1 from the other; $49 \times 1 = 49$.

The second double ordinate crosses at 45 and 5; $45 \times 5 = 225$.

The third at 41 and 9; $41 \times 9 = 369$, &c.

Set 625, the square of the semi-transverse, on C, over 40 on D; then under 49 of C is 11.2 on D, the first double ordinate; under 225 is 24, the second; under 369 is 30.735, the third, &c.

305. Required the area of a curvilineal figure, of

which the following are the dimensions:—The ordinates, 29.75 ; 56 55 ; 70.69 ; 79.26 ; 83.98 ; 85.5 ; 83.98 ; 79.26 ; 70.69 ; 56.55 ; and 29.75 ; the common distance of the ordinates, 12 ; and the versine, or height of each terminal segment, 4 inches.

ORDINATES.		4	2	1
29.75	′	56.55	70.69	29.75
56.55	′′′′	79.26	83.98	29.75
70.69	′′	85.5	83.98	59.5
79.26	′′′′	79.26	70.69	8
83.98	′′	56.55	309.64	476.0
85.5	′′′′	357.12	618.68	
83.98	′′	1428.48		
79.26	′′′′	618.68		
70.69	′′	59.5		
56.55	′′′′	2106.66		
29.75	′	12		

25279.92 thrice the interordinate areas.
476. thrice the exordinate areas.
25755.92 thrice the total area.

25755.92
.0012022
5151184
5151184
5151184
2575592
30.963767024 gallons.

306. This figure is also a true ellipse, of which the ordinates have been computed. For its area, by Problem 9, we have

.00283257
128 **transverse.**
2266056
3399084
.36256896
85.5 **conjugate.**
181284480
181284480
290055168
30.999646080 **true content.**

307. Required the area in gallons of a curvilineal plane, the ordinates of which are 70 ; 79 ; 80 ; 78.6 ; and 69 ; the common distance 24 ; and the height of each terminal segment, 8 inches.

ORDINATES.		4	2	1
70	′	79	80	70
79	′′′′	78.6	160	69
80	′′	157.6		139
78.6	′′′′	630.4		16
69	′	160		834
		139		139
		929.4		2224

24
37176
18588
22305.6
2224.
24529.6
.0012022
490592
490592
2943552
29.48948512 **gallons.**

308. Find the area, in gallons, of a curvilineal space, the ordinates of which are 61.4 ; 101.2 ; 118 ; 123.7 ; 125.2 ; 124 ; 118.6 ; 99.9 ; 57 ; their common distance, 17 ; and the versine, or height of each terminal segment, 7.85.

ORDINATES.	4	2	1
61.4 ′	101.2	118	61.4
101.2 ′′′′	123.7	125.2	57.
118. ′′	124.	118.6	118.4
123.7 ′′′′	99.9	361.8	15.7
125.2 ′′	448.8	723.6	8288
124. ′′′′	1795.2		5920
118.6 ′′	723.6		1184
99.9 ′′′′	118.4		1858.88
57 ′	2637.2		
	17		
	184604		
	26372		
	44832.4		
	1858.88		
	46691.28		
	.0012022		
	9338256		
	9338256		
	56029536		
	56.132256816 gallons.		

309. Find the area of a curvilinear plane, the ordinates of which are 82.4 ; 96.5 ; 112 ; 119.2 ;
′ ′′′′ ′′ ′′′′
121.3 ; 122.4 ; 123 ; 122.6 ; 121.2 ; 118.9 ; 112 ;
′′ ′′′′ ′′ ′′′′ ′′ ′′′′ ′′

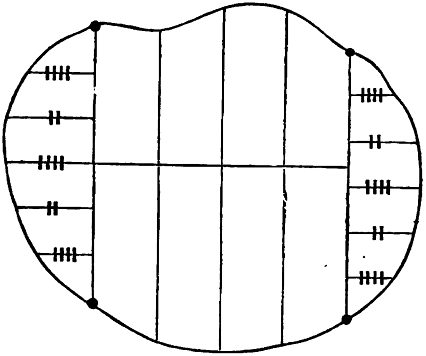
$\underset{////}{96.3}$; and $\underset{/}{82.4}$; common distance, 12 ; height of each terminal segment, 2 inches.—*Ans.* 59.18 gallons.

310. If the figure is very irregular, it will be advisable to proceed with the end areas as follows:—Having estimated the middle, or what we have called the interordinate space, turn the figure a quarter round, consider the first ordinate as a diameter, divide it into an even number of equal parts, and perpendicular to the divisions erect a number of fresh ordinates, which will, of course, be at right angles to the original ordinates drawn across the middle of the figure. The area for this is to be computed as before; only here it is to be observed that the extreme, or first and last ordinates, will each be 0. Lastly, turn the figure half round, and proceed with the right hand in the same manner.

EXAMPLE.

311. Across the middle of an oval, irregular figure, 5 ordinates were taken, as follows, $\underset{/}{84}$; $\underset{////}{97}$; $\underset{//}{112}$; $\underset{////}{104}$; and $\underset{/}{72}$ inches; with a common distance between them of 20 inches. On the left hand 7 ordinates were taken, viz., $\underset{/}{0}$; $\underset{////}{17}$; $\underset{//}{23}$; $\underset{////}{27}$; $\underset{//}{29}$; $\underset{////}{21}$; and $\underset{/}{0}$; with a common distance of 14 inches. On the right

hand, 7 ordinates, viz., $\underset{\prime}{0}$; $\underset{\prime\prime\prime\prime}{9}$; $\underset{\prime\prime}{21}$; $\underset{\prime\prime\prime\prime}{23}$; $\underset{\prime\prime}{20}$; $\underset{\prime\prime\prime\prime}{14}$; $\underset{\prime}{0}$; with a common distance of 12 inches: required the content.



For the middle space—

4	2	1
97	112	84
104	224	72
201		156
804		
224		
156		
1184		
20		
23680 thrice middle.		

For the left hand—

```
    4        2        1
   17       23        0
   27       29        0
   21       52
   65      104
  260
  104
  364
   14
 5096 thrice left.
```

For the right hand—

```
    4        2        1
    9       21        0
   23       20        0
   14       41
   46       82
  184
   82
  266
   12
 3192 thrice right.
```

```
     23680
      5096
      3192
     31968 thrice total area.
  .0012022
     63936
    63936
 383616
38.4319296 gallons.
```

312. SUMMARY OF FORMULÆ FOR AREAS.

1 $\text{Square} = \dfrac{s^2}{\text{square}}$

2 $\text{Parallelogram} = \dfrac{l\,b}{\text{square}}$

3 $\text{Triangle} = \dfrac{b\,p}{\text{triangle}}$

4 $\text{Trapezium} = \dfrac{(P + p)d}{\text{triangle}}$

5 $\text{Trapezoid} = \dfrac{(S + s)p}{\text{triangle}}$

6 $\text{Polygon} = \dfrac{s^2}{\text{polygonal prism}}$

7 $\text{Circle} = \dfrac{d^2}{\text{circle}}$

8 $\text{Sector} = \dfrac{r\,l}{\text{triangle}}$

9 $\text{Ellipse} = \dfrac{t\,c}{\text{circle}}$

10 $\text{Parabola} = \dfrac{b\,h}{\text{parabola}}$

MENSURATION OF SOLIDS.

313. Having shown the method of computing the content of areas, it will readily be perceived, that for all upright vessels, as prisms or cylinders, in which the area is uniform throughout, we have only to multiply such area by the depth of the vessel to find the total content of the vessel; to alter, in short, the formula from $\frac{1\ s^2}{\mathrm{V}}$ to $\frac{h\ s^2}{\mathrm{V}}$. For other solids, as pyramids, cones, spheres, spheroids, paraboloids, and parabolic spindles, which are in a known ratio to their circumscribing square prism, the rule for computing the content is equally easy, as we have simply to multiply the square of the side or diameter by the height, and to select the divisor, factor, or gauge point, proper for the particular kind of solid under examination. Frustums and Ungulas will then be the only figures that will require to be treated of.

Problem 1.

314. To find the content of a square prism.

Rule.—Multiply the square of the base by the height, and use the divisors, factors, and gauge points, standing under the word square prism.

FORMULÆ FOR GALLONS.

$$\frac{h\,s^2}{277.274}\ ;\ .0036065\,h\,s^2\ ;\ \frac{h\,s^2}{16.65^2}$$

EXAMPLES.

315. The sides of a square prism are each 30 inches, the depth 50 inches: required its content.

$$\frac{50 \times 30}{277.274} = 162.29 \text{ gallons.}$$

$$.0036065 \times 50 \times 30^2 = 162.29$$

By the slide-rule—

$$\frac{50 \times 30^2}{16.65^2} = ?$$

16.65 D : 50 C :: 30 D : 162.3 C

316. Required the content of a cubical vessel, each side of which is 60 inches.

$$\frac{60 \times 60^2}{277.274} = 779 \text{ gallons.}$$

$$.0036065 \times 60 \times 60^2 = 779$$

By the slide-rule—

$$\frac{60 \times 60^2}{16.65^2} = ?$$

16.65 D : 60 C :: 60 D : 779 C

317. What is the content of a square prism 16 inches deep, 15 square?—*Ans.* 12.98.

318. Required the content of a cubical vessel, each side of which measures 43.7 inches.—*Ans.* 301 gallons, nearly.

Problem 2.

319. To find the content of a parallelopipedon, or rectangular prism.

Rule.—Multiply together the length, breadth, and depth, and use the divisors, factors, and gauge points, under the word square prism.

Formulæ for Gallons.

$$\frac{l\,b\,d}{277.274}\ ;\ .0036065\,l\,b\,d\ ;\ \frac{l\,q^2}{16.65^2}$$

Examples.

320. The length of a vessel in the shape of a rectangular oblong prism is 82 inches, its breadth 26, and depth 29 inches: required its content.

$$\frac{82 \times 26 \times 29}{277.274} = 222.98 \text{ gallons.}$$

$$.0036065 \times 82 \times 26 \times 29 = 222.98$$

By the slide-rule—

$$\frac{82 \times \sqrt{26 \times 29})^2}{16.65^2} = \frac{82 \times 27.46^2}{16.65^2} = ?$$

Set the 26 of C and the 26 of D together; under 29 of C is 27.46, the mean proportional; then

16.65 D : 82 C :: 27.46 D : 223 C

321. A cistern is 72 inches long, 32 broad, and 44 deep: required the number of gallons it will contain. —*Ans.* 365.6.

Problem 3.

322. To find the content of a parabolic prism.

RULE.—Multiply the base of the parabola by its height, and by the depth of the vessel, and use the divisors, factors, and gauge points, standing under the words parabolic prism.

FORMULÆ FOR GALLONS.

$$\frac{d\,b\,h}{415.91}\ ;\ .002404\,d\,b\,h\ ;\ \frac{d\,q^2}{20.39^2}.$$

EXAMPLES.

323. The side or double ordinate of a parabolic prism is 26 inches, from the middle of which the height to the vertex, or abscissa, is 40 inches, the depth of the vessel 60: required its content.

$$\frac{60 \times 26 \times 40}{415.91} = 150 \text{ gallons.}$$

$$.002404 \times 60 \times 26 \times 40 = 150$$

By the slide-rule—

$$\frac{60 \times \sqrt{26 \times 40}^2}{20.39^2} = \frac{60 \times 32.25^2}{20.39^2} = ?$$

Set the 26 of C and the 26 of D together; under 40 of C is 32.25, the mean proportional; then

$$20.39\text{ D} : 60\text{ C} :: 32.25\text{ D} : 150\text{ C}$$

324. The side of a parabolic prism is 40 inches, the height 24, the depth 35: required its content.—*Ans.* 80.77.

Problem 4.

325. To find the content of a polygonal prism.

RULE.—Multiply the square of the side by the depth or height, and use the divisors, factors, and gauge points, standing opposite the appropriate name under the word prism.

FORMULÆ.

$$\frac{h s^2}{V} ; F h s^2 ; \frac{h s^2}{G^2}$$

EXAMPLES.

326. The side of a regular hexagonal prism is 12 inches, its depth 25: required its content.

$$\frac{25 \times 12^2}{106.72} = 33.73 \text{ gallons.}$$

$$25 \times 12^2 \times .00937 = 33.73$$

By the slide-rule—

$$\frac{25 \times 12^2}{10.33^2} = ?$$

$$10.33 \text{ D} : 25 \text{ C} :: 12 \text{ D} : 33.73 \text{ C}$$

327. The side of a regular octagonal prism is 25 inches, the depth 10: required its content.—*Ans.* 108.8 gallons.

328. Required the content of a regular pentagonal prism, the sides being each 22 inches, the depth 12 inches.—*Ans.* 36 gallons.

Problem 5.

329. To find the content of an irregular prism.

Rule.—Multiply the area of the base by the perpendicular height.

Examples.

330. The base of a cistern is a trapezium, the diagonal of which is 90 inches, the two perpendiculars to it 36 and 42, the depth of the vessel 48: required its content.

By Problem 4, Mensuration of Areas, section 226—

$$.00180325 \times \overline{36 + 42} \times 90 =$$

$$.00180325 \times 78 \times 90 = \text{the area}$$

$$\therefore\ .00180325 \times 78 \times 90 \times 48 = 607.6 \text{ gallons.}$$

331. Required the content of a triangular prism 40 inches deep; the side of the triangular base is 100 inches, the perpendicular from which to the opposite angle is 60 inches.—*Ans.* 432.78 gallons.

Problem 6.

332. To find the content of a cylinder.

Rule.—Multiply the square of the diameter by the perpendicular height or depth, and use the divisors, factors, and gauge points, standing under the word cylinder.

FORMULÆ FOR GALLONS.

$$\frac{h\,d^2}{353.036}\ ;\ .00283257\,h\,d^2\ ;\ \frac{h\,d^2}{18.78^2}$$

EXAMPLES.

333. The height, or depth, of a cylinder is 40 inches, its diameter 21.5: required its content in gallons.

$$\frac{40 \times 21.5^2}{353.036} = 52.37 \text{ gallons.}$$

$$.00283257 \times 40 \times 21.5^2 = 52.37$$

By the slide-rule—

$$\frac{40 \times 21.5^2}{18.78^2} = ?$$

18.78 D : 40 C :: 21.5 D : 52.37 C

334. Required the content of a cylinder 62.5 inches deep, with a diameter of 54.8 inches.—*Ans.* 531.64 gallons.

335. A cylindrical vessel is 30 inches deep; its diameter is also 30 inches: what is its content?—*Ans.* 76.47 gallons.

Problem 7.

336. To find the content of an elliptic cylinder.

RULE.—Multiply the product of the two diameters by the perpendicular height, or depth, and use the divisors, factors, and gauge points, under the word ellipse, or cylinder.

FORMULÆ FOR GALLONS.

$$\frac{h\,t\,c}{353.036}\;;\;.00283257\,h\,t\,c\;;\;\frac{h\,q^2}{18.78^2}$$

EXAMPLES.

337. The axes of an elliptic cylinder are 67 and 52, its depth 50 inches: required its content in gallons.

$$\frac{50 \times 67 \times 52}{353.036} = 493.4 \text{ gallons.}$$

$$.00283257 \times 50 \times 67 \times 52 = 493.4$$

By the slide-rule—

$$\frac{50 \times \sqrt{67 \times 52}^{\,2}}{18.78^2} = \frac{50 \times 59.03^2}{18.78^2} = ?$$

Set the 67 of C and the 67 of D together; under 52 of C is 59.03, the mean proportional; then

18.78 D : 50 C :: 59.03 D : 493.4 C

338. The transverse diameter of an elliptic cylinder is 40 inches, the conjugate 32, the depth 15 inches: required the content.—*Ans.* 54.38 gallons.

339. If the transverse diameter of an elliptic cylinder measures 24 inches, its conjugate 18, and its depth 12 inches: what is its content in gallons?—*Ans.* 14.68 gallons.

Problem 8.

340. To find the content of a square pyramid.

RULE.—Multiply the square of the side by the depth, and use the divisors, factors, and gauge points, under the word square pyramid.

FORMULÆ FOR GALLONS.

$$\frac{h\,s^2}{831.822}\ ;\ .0012022\,h\,s^2\ ;\ \frac{h\,s^2}{28.84^2}$$

EXAMPLES.

341. A vessel in the shape of an inverted square pyramid measures 19 inches along each side, and is 52 inches deep: required its content in gallons.

$$\frac{52 \times 19^2}{831.822} = 22.56 \text{ gallons.}$$

$$.0012022 \times 52 \times 19^2 = 22.56$$

By the slide-rule—

$$\frac{52 \times 19^2}{28.84^2} = ?$$

28.84 D : 52 C :: 19 D : 22.56 C

342. Required the content of an inverted square pyramid 93 inches deep, and 25 along each side.—*Ans.* 69.8 gallons.

Problem 9.

343. To find the content of a rectangular pyramid.

RULE.—Multiply together the length, breadth, and depth, and use the divisors, factors, and gauge points, under rectangular or square pyramid.

FORMULÆ FOR GALLONS.

$$\frac{l\,b\,d}{831.822}\ ;\ .0012022\,l\,b\,d\ ;\ \frac{l\,q^2}{28.84^2}$$

EXAMPLES.

344. A hopper, in the shape of an inverted oblong pyramid, measures at top 45 inches long and 36 broad; its perpendicular depth is 50 inches: required its content.

$$\frac{50 \times 45 \times 36}{831.822} = 97.37 \text{ gallons.}$$

$$.0012022 \times 50 \times 45 \times 36 = 97.37$$

By the slide-rule—

$$\frac{50 \times \sqrt{45 \times 36}^{\,2}}{28.84^2} = \frac{50 \times 40.25^2}{28.84^2} = ?$$

Set the 45 of C and the 45 of D together; under 36 of C is 40.25, the mean proportional; then

28.84 D : 50 C :: 40.25 D : 97.3 C

345. A vessel, oblong at top, and tapering downwards to a point, measures 48 inches by 75; its depth is 63 inches: required its content in gallons.—*Ans.* 272.65 gallons.

Problem 10.

346. To find the content of a polygonal pyramid.

RULE.—Multiply the square of the side by the perpendicular depth, and use the divisors, factors, and gauge points, standing opposite the appropriate name under the word pyramid.

FORMULÆ.

$$\frac{h\,s^2}{V}\ ;\ F\,h\,s^2\ ;\ \frac{h\,s^2}{G^2}$$

EXAMPLES.

347. Required the content of a pentagonal pyramid, each side of the top measuring 36 inches, and the depth being 25 inches.

$$\frac{25 \times 36^2}{483.48} = 67 \text{ gallons.}$$

$$.002068 \times 25 \times 36^2 = 67$$

By the slide-rule—

$$\frac{25 \times 36^2}{21.98^2} = ?$$

$$21.98\text{ D} : 25\text{ C} :: 36\text{ D} : 67\text{ C}$$

348. What is the content of a nonagonal pyramid, each side of which measures 13 inches, and the depth 28 inches?—*Ans.* 35.16 gallons.

349. Required the content of a heptagonal pyramid measuring 32 inches along each side, and 50 deep.—*Ans.* 223.7 gallons.

Problem 11.

350. To find the content of a cone.

Rule.—Multiply the square of the diameter by the perpendicular depth, and use the divisors, factors, and gauge points, under the word cone.

Formulæ for Gallons.

$$\frac{h\,d^2}{1059.108}\;;\;.0009442\,h\,d^2\;;\;\frac{h\,d^2}{32.54^2}$$

Examples.

351. The diameter of a conical vessel is 21 inches, and the depth 100: required its content in gallons.

$$\frac{100 \times 21^2}{1059.108} = 41.63 \text{ gallons.}$$

$$.0009442 \times 100 \times 21^2 = 41.63$$

By the slide-rule—

$$\frac{100 \times 21^2}{32.54^2} = ?$$

$$32.54\,\text{D} : 100\,\text{C} :: 21\,\text{D} : 41.63\,\text{C}$$

352. Required the content of a cone whose diameter is 46 inches, and perpendicular depth 39 inches.—*Ans*. 77.9 gallons.

Problem 12.

353. To find the content of an elliptical cone.

Rule.—Multiply the product of the diameters by the perpendicular depth, and use the divisors, factors, and gauge points, standing under the words circular or elliptic cone.

Formulæ for Gallons.

$$\frac{htc}{1059.108}\ ;\ .0009442\,htc\ ;\ \frac{hq^2}{32.54^2}$$

Examples.

354. The axes at the top of an inverted elliptic cone are 40 and 32 inches; its perpendicular depth is 18 inches: required its content.

$$\frac{18 \times 40 \times 32}{1059.108} = 21.75 \text{ gallons.}$$

$$.0009442 \times 18 \times 40 \times 32$$

By the slide-rule—

$$\frac{18\,\sqrt{40 \times 32})^2}{32.54^2} = \frac{18 \times 35.78^2}{32.54^2} = ?$$

Place the 40 of C and the 40 of D together; under 32 of C is 35.78, the mean proportional; then

32.54 D : 18 C :: 35.78 D : 21.75 C

355. A cone is 24 inches deep; the major axis is 12 inches, the minor 10: required the number of gallons it will contain.—*Ans.* 2.7 gallons.

356. The axes of an inverted elliptic cone are 16 and 9 inches, the depth 19 inches: required the content.—*Ans.* 2.58 gallons.

Problem 13.

357. To find the content of a sphere or spheroid.

Rule.—Multiply the fixed axis by the square of the revolving one, and use the divisors, factors, and gauge points, under the word sphere or spheroid.

FORMULÆ FOR GALLONS.

For spheres—

$$\frac{d\,d^2}{529.554}\ ;\ .0018884\,d\,d^2\ ;\ \frac{d\,d^2}{23^2}$$

For spheroids—

$$\frac{f\,v^2}{529.554}\ ;\ .0018884\,f\,v^2\ ;\ \frac{f\,v^2}{23^2}$$

EXAMPLES.

358. The diameter of a sphere is 25 inches: required its content in gallons.

$$\frac{25 \times 25^2}{529.554} = 29.5 \text{ gallons.}$$

$$.0018884 \times 25 \times 25^2 = 29.5$$

By the slide-rule—

$$\frac{25 \times 25^2}{23^2} = ?$$

$$23\ \text{D} : 25\ \text{C} :: 25\ \text{D} : 29.5\ \text{C}$$

359. Required the content of a prolate spheroid, the revolving axis of which is 36 inches, and the fixed axis 54.

$$\frac{54 \times 36^2}{529.554} = 132.15 \text{ gallons.}$$

$$.0018884 \times 54 \times 36^2 = 132.15$$

By the slide-rule—

$$\frac{54 \times 36^2}{23^2} = ?$$

$$23 \text{ D} : 54 \text{ C} :: 36 \text{ D} : 132 \text{ C}$$

360. Required the content of an oblate spheroid, the fixed axis being 36, and the revolving axis 54 inches.

$$\frac{36 \times 54^2}{529.554} = 198.23 \text{ gallons.}$$

$$.0018884 \times 36 \times 54^2 = 198.23$$

By the slide-rule—

$$\frac{36 \times 54^2}{23^2} = ?$$

$$23 \text{ D} : 36 \text{ C} :: 54 \text{ D} : 198.2 \text{ C}$$

As the axes of these two spheroids have here shifted places, the content of the former to the latter is, of course, as 36 to 54, or as 2 : 3. Hence, under 2 of A set the content of the first; then under 3 is the content of the second.

361. Required the content of a sphere whose diameter is 20 inches.—*Ans.* 15.1 gallons.

362. The diameter of a spherical vessel is 52 inches: required its content.—*Ans.* 265.5 gallons.

363. Required the content of a prolate spheroid, the transverse or fixed axis of which is 54 inches, and the conjugate or revolving axis 33 inches.—*Ans.* 111 gallons.

364. Required the content of an oblate spheroid, he fixed or minor axis being 33, and the revolving or major axis 54 inches.—*Ans.* 181.7 gallons.

These last two spheroids are to each other as 33 to 54, or as 11 : 18.

Problem 14.

365. To find the content of a paraboloid.

Rule.—Multiply the square of the diameter by the height, and use the divisors, factors, and gauge points, under the word paraboloid.

FORMULÆ FOR GALLONS.

$$\frac{h\,d^2}{706.072}\,;\ .00141628\,h\,d^2\,;\ \frac{h\,d^2}{26.57^2}$$

EXAMPLES.

366. The diameter of the top of a vessel in the form of a paraboloid is 82 inches, and the depth 96 inches: required the number of gallons it will contain.

$$\frac{96 \times 82^2}{706.072} = 914.2 \text{ gallons.}$$

$$.00141628 \times 96 \times 82^2 = 914.2$$

By the slide-rule—

$$\frac{96 \times 82^2}{26.57^2} = ?$$

$$26.57 \text{ D} : 96 \text{ C} :: 82 \text{ D} : 914 \text{ C}$$

367. The height of a vessel in the form of a parabolic conoid is 42 inches, and the diameter 24 inches: what is the content in gallons?—*Ans.* 34.25 gallons.

Problem 15.

368. To find the content of a parabolic spindle.

Rule.—Multiply the square of the middle diameter by the length of the spindle, and use the divisors, factors, and gauge points, standing under the words parabolic spindle.

FORMULÆ FOR GALLONS.

$$\frac{l\,d^2}{661.94}\ ;\ .0015107\,l\,d^2\ ;\ \frac{l\,d^2}{25.72^2}$$

EXAMPLES.

369. The length of a parabolic spindle is 70 inches, and its middle diameter 28 inches: require the content in gallons.

$$\frac{70 \times 28^2}{661.94} = 82.9 \text{ gallons.}$$

$$.0015107 \times 70 \times 28^2 = 82.9$$

By the slide-rule—

$$\frac{70 \times 28^2}{25.72^2} = ?$$

$$25.72 \text{ D} : 70 \text{ C} :: 28 \text{ D} : 82.9 \text{ C}$$

370. The length of a vessel in the form of a parabolic spindle is 65 inches, and its middle diameter 24 inches: required its content in gallons.—*Ans.* 56.56.

371. The length of a parabolic spindle is 60 inches, and the middle diameter 34 inches: required the solidity of the spindle in cubic feet.—*Ans.* 16.79.

Problem 16.

372. To find the content of an hyperbolic conoid, or hyperboloid.

Rule 1st.—Add together the square of the top diameter, and the square of double the intermediate diameter, multiply by the depth or height, and use the divisors, factors, and gauge points, under the words round frustum.

FORMULÆ FOR GALLONS.

$$\frac{h(D^2 + \overline{2\,\delta}^2)}{2118.216} \ ; \ .0004721\, h(D^2 + \overline{2\,\delta}^2)$$

$$\frac{h(D^2 + \overline{2\,\delta}^2)}{46^2}$$

373. Required the content of an hyperboloid 50 inches deep, the top diameter being 52 inches, and the intermediate diameter, or diameter at half the depth, 34 inches.

$$\begin{array}{lr}
\text{Top diameter} & \left.\begin{array}{r}52\\68\end{array}\right|^2 \begin{array}{r}= 2704\\= 4624\end{array}\\
\text{Twice intermediate} & \\
 & \underline{}\\
 & 7328\\
 & \underline{}
\end{array}$$

$$7328 \times 50 \div 2118.216 = 172.977 \text{ gallons.}$$
$$7328 \times 50 \times .0004721 = 172.977$$

By the slide-rule—

$$\frac{50}{46^2}\left\{\begin{array}{r}52\\68\end{array}\right|^2 \begin{array}{r}= 63.8\\= 109.1\\\hline 172.9\\\hline\end{array}$$

$$\begin{array}{ccccccc}
\text{D} & & \text{C} & & \text{D} & & \text{C}\\
46 & : & 50 & :: & 52 & : & 63.8\\
 & & & :: & 68 & : & 109.1\\
\cline{7-7}
 & & & & & & 172.9\\
\cline{7-7}
\end{array}$$

374. What is the content of an hyperboloid 24 inches deep, the top diameter being 30 inches, the intermediate 18 inches?—*Ans*. 24.88.

375. Rule 2nd.—Add together the square of the

top radius and the square of the intermediate diameter, multiply by the depth or height, and use the divisors, factors, and gauge points, under the word sphere or spheroid.

Formulæ for Gallons.

$$\frac{h(R^2 + \delta^2)}{529.554} \; ; \; .0018884\, h(R^2 + \delta^2)$$

$$\frac{h(R^2 + \delta^2)}{23^2}$$

Examples.

376. Required the content of an hyperbolic conoid 50 inches deep, the radius at the top being 26 inches, and the intermediate diameter 34 inches.

$$\begin{array}{ll} \text{Top radius . . . .} & \left.\begin{array}{c} 26 \\ 34 \end{array}\right|^2 \begin{array}{r} = \;\;676 \\ = 1156 \\ \hline 1832 \\ \hline \end{array} \\ \text{Intermediate diameter} & \end{array}$$

$1832 \times 50 \div 529.554 = 172.977$, as before.

$1832 \times 50 \times .0018884 = 172.977$

By the slide-rule—

$$\frac{50}{23^2}\left\{\left.\begin{array}{c} 26 \\ 34 \end{array}\right|^2\right. \begin{array}{r} = \;\;63.8 \\ = 109.1 \\ \hline 172.9 \\ \hline \end{array}$$

$$\begin{array}{ccccccc} D & & C & & D & & C \\ 23 & : & 50 & :: & 26 & : & 63.8 \\ & & & :: & 34 & : & 109.1 \\ \hline & & & & & & 172.9 \end{array}$$

Problem 17.

377. To find the content of spindles in general by the intermediate diameter.

Rule 1st.—Add together the square of the middle diameter and the square of double the intermediate diameter, multiply by the length, and use the divisors, factors, and gauge points, under the words round frustum.

Formulæ for Gallons.

$$\frac{l(D^2 + \overline{2\,\delta}|^2)}{2118.216} \; ; \; .0004721\, l(D^2 + \overline{2\,\delta}|^2)$$

$$\frac{l(D^2 + \overline{2\,\delta}|^2)}{46^2}$$

378. Required the content of a spindle 70 inches long, the middle diameter of which is 28 inches, and the intermediate 20 inches.

$$\begin{array}{lrcr} \text{Middle diameter} & 28 & \rceil^2 = & 784 \\ \text{Twice intermediate} & 40 & | = & 1600 \\ \hline & & & 2384 \end{array}$$

2384 × 70 ÷ 2118.216 = 78.78 gallons.
2384 × 70 × .0004721 = 78.78

By the slide-rule—

$$\frac{70}{46^2}\left\{\begin{array}{c} 28 \\ 40 \end{array}\right|^2 \begin{array}{l} = 25.9 \\ = 52.9 \\ \hline 78.8 \\ \hline \end{array}$$

$$\begin{array}{ccccccc} \text{D} & & \text{C} & & \text{D} & & \text{C} \\ 46 & : & 70 & :: & 28 & : & 25.9 \\ & & & :: & 40 & : & 52.9 \\ \cline{7-7} & & & & & & 78.8 \\ \cline{7-7} \end{array}$$

379. What is the content of a spindle whose length is 40 inches, the middle diameter 12, and the intermediate diameter 9.5 inches?—*Ans.* 9.536 gallons.

380. Rule 2nd.—Add together the square of the middle radius and the square of the intermediate diameter, multiply by the length, and use the divisors, factors, and gauge points, under the word sphere or spheroid.

Formulæ for Gallons.

$$\frac{l(R^2 + \delta^2)}{529.554} \;;\; .0018884\, l(R^2 + \delta^2)$$

$$\frac{l(R^2 + \delta^2)}{23^2}$$

381. Required the content of a spindle 70 inches long, the middle radius of which is 14 inches, and the intermediate diameter 20 inches.

$$\begin{array}{lll} \text{Middle radius} \cdot & \left.\begin{array}{c} \overline{14} \\ 20 \end{array}\right]^2 & \begin{array}{r} = 196 \\ = 400 \\ \hline 596 \\ \hline \end{array} \\ \text{Intermediate diameter} & & \end{array}$$

$596 \times 70 \div 529.554 = 78.78$, as before.

$596 \times 70 \times .0018884 = 78.78$

By the slide-rule—

$$\frac{70}{23^2}\left\{\left.\begin{array}{c} \overline{14} \\ 20 \end{array}\right]^2\right. \begin{array}{r} = 25.9 \\ = 52.9 \\ \hline 78.8 \\ \hline \end{array}$$

$$\begin{array}{ccccccc} D & & C & & D & & C \\ 23 & : & 70 & :: & 14 & : & 25.9 \\ & & & :: & 20 & : & 52.9 \\ \hline & & & & & & 78.8 \\ \hline \end{array}$$

Problem 18.

382. To find the content of the frustum of a pyramid.

Rule.—Add together the square of the side of the base, the square of the side of the top, and the square of their sum, multiply by the length, and use the divisors, factors, and gauge points, under the word square or polygonal frustum, according to the pyramid of which the frustum is a portion.

FORMULÆ FOR GALLONS.

For frustum of square pyramid—

$$\frac{l(s^2 + S^2 + \overline{s + S}|^2)}{1663.644} = \text{gallons.}$$

$$.0006011\, l(s^2 + S^2 + \overline{s + S}|^2)$$

$$\frac{l(s^2 + S^2 + \overline{s + S}|^2)}{40.79^2}$$

For frustum of polygonal pyramid—

$$\frac{l(s^2 + S^2 + \overline{s + S}|^2)}{V}\ ;\ F\, l(s^2 + S^2 + \overline{s + S}|^2);$$

$$\frac{l(s^2 + S^2 + \overline{s + S}|^2)}{G^2}$$

EXAMPLES.

383. Required the content of the frustum of a square pyramid, each side of the base being 16 inches, of the top 22 inches, and the depth of the frustum 20.

$$\begin{array}{lr|l}
\text{Top} & 22 & ^2 = \ \ 484 \\
\text{Bottom} & 16 & \ \ = \ \ 256 \\
\text{Sum} & 38 & \ \ = 1444 \\
\hline
 & & 2184 \times 20 \div 1663.644 = 26.25 \text{ gallons.}
\end{array}$$

$$2184 \times 20 \times .0006011 = 26.25$$

By the slide-rule—

$$\frac{20}{40.79^2}\left\{\left.\begin{array}{c} 22 \\ 16 \\ 38 \end{array}\right)^2\right. \begin{array}{r} = 5.8 \\ = 3.08 \\ = 17.36 \\ \hline 26.24 \end{array}$$

$$\begin{array}{rcrcrcr} \text{D} & & \text{C} & & \text{D} & & \text{C} \\ 40.79 & : & 20 & :: & 22 & : & 5.8 \\ & & & :: & 16 & : & 3.08 \\ & & & :: & 38 & : & 17.36 \\ \hline & & & & & & 26.24 \end{array}$$

384. The side of the larger end of a square pyramidical guile-tun is 42.5 inches, of the smaller 30 inches, and the depth is 36: required its content.

$$\left.\begin{array}{ll} \text{Larger} & 42.5 \\ \text{Smaller} & 30 \\ \text{Sum} & 72.5 \end{array}\right)^2 \begin{array}{r} = 1806.25 \\ = 900. \\ = 5256.25 \\ \hline 7962.5 \end{array}$$

$7962.5 \times 36 \div 1663.644 = 172.3$ gallons.

$7962.5 \times 36 \times .0006011 = 172.3$

By the slide-rule—

$$\frac{36}{40.79^2}\left\{\left.\begin{array}{c} 42.5 \\ 30. \\ 72.5 \end{array}\right)^2\right. \begin{array}{r} = 39 \\ = 19.3 \\ = 116. \\ \hline 172.3 \end{array}$$

$$\begin{array}{llllllll}
 & \text{D} & & \text{C} & & \text{D} & & \text{C} \\
 & 40.79 & : & 36 & :: & 42.5 & : & 39 \\
 & & & & :: & 30 & : & 19.3 \\
 & & & & :: & 72.5 & : & 116. \\
\cline{8-8}
 & & & & & & & 172.3 \\
\cline{8-8}
\end{array}$$

385. Required the content of the frustum of a heptagonal pyramid, each side of the top end being 35 inches, of the bottom 28 inches, the perpendicular depth being 30 inches.

$$\begin{array}{lll}
\text{Top} & \left.\begin{array}{r} 35 \\ 28 \\ 63 \end{array}\right)^2 & \begin{array}{r} = 1225 \\ = 784 \\ = 3969 \\ \hline 5978 \\ \hline \end{array} \\
\text{Bottom} & & \\
\text{Sum} & &
\end{array}$$

$5978 \times 30 \div 458.34 = 391.67$ gallons.

$5978 \times 30 \times .002184 = 391.67$

By the slide-rule—

$$\frac{30}{21.4^2}\left\{\left.\begin{array}{r} 35 \\ 28 \\ 63 \end{array}\right)^2 \begin{array}{r} = 80.5 \\ = 51.5 \\ = 259.7 \\ \hline 391.7 \\ \hline \end{array}\right.$$

$$\begin{array}{rcrcrcr}
\text{D} & & \text{C} & & \text{D} & & \text{C} \\
21.4 & : & 30 & :: & 35 & : & 80.5 \\
 & & & :: & 28 & : & 51.5 \\
 & & & :: & 63 & : & 259.7 \\
\hline
 & & & & & & 391.7
\end{array}$$

386. What is the content in gallons of the frustum of a square pyramid, each side of the greater end being 28 inches, of the less 16, and the depth 49 inches?—*Ans.* 87.65.

387. Required the content of the frustum of an octagonal pyramid, each side of the greater end being 35 inches, of the less 28, and the perpendicular depth 94 inches.—*Ans.* 1630.7 gallons.

Problem 19.

388. To find the content of the frustum of a cone.

Rule 1st.—Add together the square of the top diameter, the square of the bottom diameter, and the square of their sum, multiply by the length, and use the divisors, factors, and gauge points, under the words round frustum.

FORMULÆ FOR GALLONS.

$$\frac{l(d^2 + D^2 + \overline{d + D}^2)}{2118.216}$$

$$.0004721\, l(d^2 + D^2 + \overline{d + D}^2)$$

$$\frac{l(d^2 + D^2 + \overline{d + D}^2)}{46^2}$$

EXAMPLES.

389. Required the content of the frustum of a cone, the top diameter being 20 inches, the bottom 36, and the depth 43 inches.

$$\begin{array}{ll} \text{Top} & \left.\begin{array}{r} 20 \\ 36 \\ 56 \end{array}\right)^2 \\ \text{Bottom} & \\ \text{Sum} & \end{array} \quad \begin{array}{r} 400 \\ = 1296 \\ = 3136 \\ \hline 4832 \\ \hline \end{array}$$

$$4832 \times 43 \div 2118.216 = 98.09 \text{ gallons.}$$

$$4832 \times 43 \times .0004721 = 98.09$$

By the slide-rule—

$$\frac{43}{46^2}\left\{\begin{array}{r} 20 \\ 36 \\ 56 \end{array}\right)^2 \begin{array}{r} = 8.1 \\ = 26.3 \\ = 63.7 \\ \hline 98.1 \\ \hline \end{array}$$

$$\begin{array}{ccccccc} \text{D} & & \text{C} & & \text{D} & & \text{C} \\ 46 & : & 43 & :: & 20 & : & 8.1 \\ & & & :: & 36 & : & 26.3 \\ & & & :: & 56 & : & 63.7 \\ \cline{7-7} & & & & & & 98.1 \\ \cline{7-7} \end{array}$$

390. Required the content, in gallons, of the frustum of a cone, whose top diameter is 20 inches, bottom 18, and depth 26.—*Ans.* 26.72.

391. **Rule 2nd.**—Add together the square of the top radius, the square of the bottom radius, and the square of their sum, multiply by the length, and use the divisors, factors, and gauge points, under the word sphere or spheroid.

Formulæ for Gallons.

$$\frac{l(r^2 + R^2 + \overline{r + R}|^2)}{529.554}$$

$$.0018884\, l(r^2 + R^2 + \overline{r + R}|^2)$$

$$\frac{l(r^2 + R^2 + \overline{r + R}|^2)}{23^2}$$

Examples.

392. Required the content of the frustum of a cone, the top radius being 10 inches, the bottom 18, and the depth 43 inches.

$$\begin{array}{lr|l}
\text{Top} & \overline{10} & ^2 = 100 \\
\text{Bottom} & 18 & = 324 \\
\text{Sum} & 28 & = 784 \\
\hline
 & & \quad 1208
\end{array}$$

1208 × 43 ÷ 529.554 = 98.09, as before.

1208 × 43 × .0018884 = 98.09

By the slide-rule—

$$\frac{43}{23^2}\left\{\begin{array}{r|l}
\overline{10} & ^2 = 8.1 \\
18 & = 26.3 \\
28 & = 63.7 \\
\hline
 & \quad 98.1
\end{array}\right.$$

$$\begin{array}{cccccccc} D & & C & & D & & C \\ 23 & : & 43 & :: & 10 & : & 8.1 \\ & & & :: & 18 & : & 26.3 \\ & & & :: & 28 & : & \underline{63.7} \\ & & & & & & \underline{98.1} \end{array}$$

393. Required the content, in gallons, of the frustum of a cone, whose top radius is 10 inches, bottom 9, and depth 26.—*Ans.* 26.72.

Problem 20.

394. To find the content of the frustum of a paraboloid.

Rule.—Add together the square of the top diameter and the square of the bottom diameter, multiply by the height, and use the divisors, factors, and gauge points, under the word paraboloid.

FORMULÆ FOR GALLONS.

$$\frac{h(d^2 + D^2)}{706.072}\ ;\ .00141628\, h(d^2 + D^2)$$

$$\frac{h(d^2 + D^2)}{26.57^2}$$

395. The top and bottom diameters of the frustum of a paraboloid are 30 and 58 inches, the height 9 inches: required its content.

$$\begin{array}{lr|l}
\text{Top diameter} & 30 & ^2 = 900 \\
\text{Bottom} \quad . \quad . & 58 & = 3364 \\
\hline
 & & 4264
\end{array}$$

$$4264 \times 9 \div 706.072 = 54.35 \text{ gallons.}$$

$$4264 \times 9 \times .00141628 = 54.35$$

By the slide-rule—

$$\frac{9}{26.57^2}\left\{\begin{array}{c|l} 30 & ^2 = 11.4 \\ 58 & = 43. \\ \hline & 54.4 \end{array}\right.$$

$$\begin{array}{ccccccc}
\text{D} & & \text{C} & & \text{D} & & \text{C} \\
26.57 & : & 9 & :: & 30 & : & 11.4 \\
 & & & :: & 58 & : & 43. \\
\hline
 & & & & & & 54.4
\end{array}$$

396. The top and bottom diameters of the frustum of a paraboloid are 24 and 32 inches, the depth is 20 inches: required the content in gallons.—*Ans.* 45.32.

Problem 21.

397. To find the content of the frustum of an hyperboloid.

Rule 1st.—Add together the square of the top diameter, the square of the bottom diameter, and the square of double the intermediate diameter, multiply by the height, and use the divisors, factors, and gauge points, under the words round frustum.

FORMULÆ FOR GALLONS.

$$\frac{h(d^2 + D^2 + \overline{2\,\delta}|^2)}{2118.216}\ ;\ 0004721\ h(d^2 + D^2 + \overline{2\,\delta}|^2)$$

$$\frac{h(d^2 + D^2 + \overline{2\,\delta}|^2)}{46^2}$$

398. How many gallons will be contained in the frustum of an hyperbolic conoid, the top and bottom diameters of which are 23 and 40 inches, the intermediate diameter 36, and depth 20 inches?

$$\left.\begin{array}{r} \overline{23} \\ 40 \\ 72 \end{array}\right|^2 \begin{array}{r} = \ \ 529 \\ = 1600 \\ = 5184 \\ \hline 7313 \\ \hline \end{array}$$

$$7313 \times 20 \div 2118.216 = 69 \text{ gallons.}$$
$$7313 \times 20 \times .0004721 = 69$$

By the slide-rule—

$$\frac{20}{46^2}\left\{\left.\begin{array}{r} \overline{23} \\ 40 \\ 72 \end{array}\right|^2\right. \begin{array}{r} = \ \ 5. \\ = 15.1 \\ = 48.9 \\ \hline 69. \\ \hline \end{array}$$

$$\begin{array}{rcrcrcr} D & & C & & D & & C \\ 46 & : & 20 & :: & 23 & : & 5. \\ & & & :: & 40 & : & 15.1 \\ & & & :: & 72 & : & 48.9 \\ \cline{7-7} & & & & & & 69. \\ \cline{7-7} \end{array}$$

399. Required the content, in gallons, of the frustum of an hyperboloid, the top and bottom diameters of which are 20 and 36 inches, the intermediate diameter 33, and the depth 18 inches.—*Ans.* 51.428 gallons.

400. Rule 2nd.—Add together the square of the top radius, the square of the bottom radius, and the square of the intermediate diameter, multiply by the height, and use the divisors, factors, and gauge points, standing under the word sphere or spheroid.

Formulæ for Gallons.

$$\frac{h(r^2 + R^2 + \delta^2)}{529.554}\ ;\ .0018884\,h(r^2 + R^2 + \delta^2)$$

$$\frac{h(r^2 + R^2 + \delta^2)}{23^2}$$

401. Required the content of the frustum of an hyperboloid, the radius at the top being 11.5, at the bottom 20 inches, the intermediate diameter 36, and the depth 20 inches.

$$\left.\begin{array}{l}\overline{11.5}\\ 20.\\ 36.\end{array}\right|^{2}\begin{array}{r}= 132.25\\ = 400.\\ = 1296.\\ \hline 1828.25\\ \hline\end{array}$$

$1828.25 \times 20 \div 529.554 = 69$, as before.

$1828.25 \times 20 \times .0018884 = 69$

By the slide-rule—

$$\frac{20}{23^2}\left\{\begin{array}{l}\overline{11.5}\\ 20.\\ 36.\end{array}\right|^{2}\begin{array}{r}= 5.\\ = 15.1\\ = 48.9\\ \hline 69.\\ \hline\end{array}$$

$$\begin{array}{ccccccc} D & & C & & D & & C\\ 23 & : & 20 & :: & 11.5 & : & 5.\\ & & & & 20. & : & 15.1\\ & & & & 36. & : & 48.9\\ \cline{7-7} & & & & & & 69.\\ \cline{7-7}\end{array}$$

Problem 22.

402. To find the content of the middle frustum of a parabolic spindle.

RULE 1ST.—Add together twice the square of the middle diameter and the square of the end diameter, from the sum subtract $\frac{1}{10}$ of the square of twice the difference of the diameters, multiply the remainder by the length, and use the divisors, factors, and gauge points, under the word cone.

FORMULÆ FOR GALLONS.

$$\frac{L(2.D^2 + d^2 - \frac{1}{10} \text{ of } \overline{.2 \text{ diff.}}|^2)}{1059.108}$$

$$.0009442\ L(2.D^2 + d^2 - \tfrac{1}{10} \text{ of } \overline{.2 \text{ diff.}}|^2)$$

$$\frac{L(2.D^2 + d^2 - \frac{1}{10} \text{ of } \overline{.2 \text{ diff.}}|^2)}{32.54^2}$$

EXAMPLES.

403. The length of a vessel in the form of the middle frustum of a parabolic spindle is 20 inches, the middle diameter is 16 inches, and the end 12: what is the content in gallons?

$$\begin{array}{rr|lr}
 & \overline{16}\ {}^2 & & = 256 \\
 & \cdot & & = 256 \\
 & 12 & & = 144 \\
 \hline
 {}^*\text{diff.} & 4 & & 656 \\
 \hline
 2 \text{ diff.} & 8 & = 64\ ;\ \frac{1}{10} & = 6.4 \\
 \hline
 & & & 649.6
\end{array}$$

649.6 × 20 ÷ 1059.108 = 12.26 gallons.
649.6 × 20 × .0009442 = 12.26

---

* The 12 subtracted from the 16 leaves 4, the difference. A line is placed above and below it, to indicate that it is to be kept distinct from the other part of the formula, being introduced simply for the sake of facilitating the computation.

By the slide-rule—

$$\begin{array}{r|c|l}
 & \overline{16}^2 & = 4.83 \\
 & \cdot & = 4.83 \\
\frac{20}{32.54^2} & \underline{12} & = \underline{2.72} \\
 & \underline{4} & \ 12.38 \\
\frac{1}{10} & 8 & = \underline{.12} \\
 & & \ \underline{12.26}
\end{array}$$

$$\begin{array}{cccccccccc}
D & & C & & D & & & & & C \\
32.54 & : & 20 & :: & 16 & & & & : & 4.83 \\
 & & & & \cdot & & & & : & 4.83 \\
 & & & :: & \underline{12} & & & & : & \underline{2.72} \\
 & & & & \underline{4} & & C & & & 12.38 \\
 & & & :: & 8 & : & 1.2 & ;\ \frac{1}{10} = & & \underline{.12} \\
 & & & & & & & & & \underline{12.26}
\end{array}$$

404. The length of a cask in the form of the middle frustum of a parabolic spindle is 30 inches, the head diameter is 22, and the bung 27 inches: required the content.—*Ans.* 54.72 gallons.

405. Rule 2nd.—Add together the square of the middle diameter and twice the square of the end radius, from the sum subtract $\frac{1}{5}$ of the square of the difference of the diameters, multiply by the length, and use the divisors, factors, and gauge points, standing under the word sphere or spheroid.

FORMULÆ FOR GALLONS.

$$\frac{l(D^2 + 2.r^2 - \frac{1}{5} \text{ of } .\overline{D - d}|^2)}{529.554}$$

$$.0018884\, l(D^2 + 2.r^2 - \frac{1}{5} \text{ of } .\overline{D - d}|^2)$$

$$\frac{l(D^2 + 2.r^2 - \frac{1}{5} \text{ of } .\overline{D - d}|^2)}{23^2}$$

EXAMPLES.

406. The length of a vessel in the form of the middle frustum of a parabolic spindle is 20 inches, the middle diameter is 16 inches, the end radius 6: required the content in gallons.

$$\begin{array}{r|lr}
\overline{16}|^2 & = & 256 \\
6 & = & 36 \\
. & = & 36 \\
\cline{1-1}\cline{3-3}
16 & & 328 \\
12 & & \\
\cline{1-1}
4 & = 16\ ;\ \frac{1}{5} = & 3.2 \\
\cline{3-3}
 & & 324.8 \\
\cline{3-3}
\end{array}$$

$$324.8 \times 20 \div 529.554 = 12.26, \text{ as before.}$$

$$324.8 \times 20 \times .0018884 = 12.26$$

By the slide rule—

$$\begin{array}{r|c|r}
 & \overline{16}^2 & = 9.66 \\
 & 6 & = 1.36 \\
 & \cdot & = 1.36 \\
\cline{2-3}
\frac{20}{23^2} & 16 & 12.38 \\
 & 12 & \\
\cline{2-2}
\frac{1}{5} & 4 & = \ \ .12 \\
\cline{3-3}
 & & 12.26 \\
\cline{3-3}
\end{array}$$

$$\begin{array}{ccccccr}
D & & C & & D & & C \\
23 & : & 20 & :: & 16 & & : 9.66 \\
 & & & :: & 6 & & : 1.36 \\
 & & & & & & : 1.36 \\
\cline{7-7}
 & & & & & & 12.38 \\
 & & & & & C & \\
 & & & :: & 4 & : .6\ ;\ \frac{1}{5} = & .12 \\
\cline{7-7}
 & & & & & & 12.26 \text{ gallons.} \\
\cline{7-7}
\end{array}$$

407. The length of a cask in the form of the middle frustum of a parabolic spindle is 30 inches, the head diameter is 22, and the bung 27 inches: required the content.—*Ans.* 54.72.

Problem 23.

408. To find the content of the middle frustum of spindles in general, by the intermediate diameter.

Rule 1st.—Add together the square of the middle diameter, the square of the end diameter, and the square of double the intermediate diameter, multiply the sum by the length, and use the divisors, factors, and gauge points, under the words round frustum.

Formulæ for Gallons.

$$\frac{l(d^2 + D^2 + \overline{2\,\delta}^2)}{2118.216}\ ;\ .0004721\, l(d^2 + D^2 + \overline{2\,\delta}^2)$$

$$\frac{l(d^2 + D^2 + \overline{2\,\delta}^2)}{46^2}$$

409. A cask in the form of the middle frustum of a spindle is 40 inches long, the bung diameter is 36 inches, the head 30, and the intermediate diameter 33.9: required its content in gallons.

$$\left.\begin{array}{l} 36 \\ 30 \\ 67.8 \end{array}\right|^2 \begin{array}{l} = 1296 \\ = 900 \\ = 4596.84 \\ \hline \ 6792.84 \end{array}$$

$6792.84 \times 40 \div 2118.216 = 128.27$ gallons.

$6792.84 \times 40 \times .0004721 = 128.27$

By the slide-rule—

$$\frac{40}{46^2}\left\{\begin{array}{c}36\\30\\67.8\end{array}\right]^2\begin{array}{l}=24.5\\=17.\\=86.8\\\hline \;\;128.3\\\hline\end{array}$$

$$\begin{array}{ccccccc}
D & & C & & D & & C\\
46 & : & 40 & :: & 36 & : & 24.5\\
 & & & :: & 30 & : & 17.\\
 & & & :: & 67.8 & : & 86.8\\
\cline{7-7}
 & & & & & & 128.3\\
\cline{7-7}
\end{array}$$

410. A cask is 40 inches long, the head diameter is 24, the bung 32, the intermediate diameter 30: required the content.—*Ans.* 98.19 gallons.

411. RULE 2ND.—Add together the square of the end radius, the square of the middle radius, and the square of the intermediate diameter, multiply the sum by the length, and use the divisors, factors, and gauge points, under the words sphere or spheroid.

FORMULÆ FOR GALLONS.

$$\frac{l(r^2+R^2+\delta^2)}{529.554}\ ;\ .0018884\,l(r^2+R^2+\delta^2)$$

$$\frac{l(r^2+R^2+\delta^2)}{23^2}$$

412. A cask in the form of the middle frustum of a spindle is 40 inches long, the radius of the head 12,

of the bung 16, the intermediate diameter 30: required the content.

$$\left.\begin{array}{r}\overline{12}\\ 16\\ 30\end{array}\right|^{2}\begin{array}{l}= 144\\ = 256\\ = 900\end{array}$$

$$\underline{\overline{1300}}$$

$1300 \times 40 \div 529.554 = 98.19$, as before.

$1300 \times 40 \times .0018884 = 98.19$

By the slide-rule—

$$\frac{40}{23^2}\left\{\begin{array}{r}\overline{12}\\ 16\\ 30\end{array}\right|^{2}\begin{array}{l}= 10.8\\ = 19.4\\ = 68.\end{array}$$

$$\underline{\overline{98.2}}$$

Problem 24.

413. To find the content of the middle frustum of a sphere or spheroid when the ends are circular, or perpendicular to the fixed axe.

RULE 1ST.—Add together twice the square of the middle diameter and the square of the end diameter, multiply the sum by the length, and use the divisors, factors, and gauge points, under the word cone.

FORMULÆ FOR GALLONS.

$$\frac{L(2.D^2 + d^2)}{1059.108}\ ;\ .0009442\ L(2.D^2 + d^2)$$

$$\frac{L(2.D^2 + d^2)}{32.54^2}$$

EXAMPLES.

414. A cask in the form of the middle frustum of a prolate spheroid is 40 inches long, the bung diameter is 36, and the head 27 inches: required the content in gallons.

$$\left.\begin{array}{r} 36 \\ \cdot \\ 27 \end{array}\right|^2 \begin{array}{r} = 1296 \\ = 1296 \\ = 729 \\ \hline 3321 \\ \hline \end{array}$$

3321 × 40 ÷ 1059.108 = 125.4 gallons.

3321 × 40 × .0009442 = 125.4

By the slide-rule—

$$\frac{40}{32.54^2}\left\{\left.\begin{array}{r} 36 \\ \cdot \\ 27 \end{array}\right|^2\right. \begin{array}{r} = 49 \\ = 49 \\ = 27.4 \\ \hline 125.4 \\ \hline \end{array}$$

$$\begin{array}{rcrcrcr}
\text{D} & & \text{C} & & \text{D} & & \text{C} \\
32.54 & : & 40 & :: & 36 & : & 49 \\
 & & & & & : & 49 \\
 & & & :: & 27 & : & 27.4 \\
\cline{7-7}
 & & & & & & 125.4 \\
\cline{7-7}
\end{array}$$

415. A cask in the form of the middle frustum of a prolate spheroid is 40 inches long, the bung diameter is 32, and the head 24: required the content.—*Ans.* 99.1 gallons.*

416. Rule 2nd.—Add together twice the square of the end radius, and the square of the middle diameter, multiply by the length, and use the divisors, factors, and gauge points, standing under the word sphere or spheroid.

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* When the ends are parallel to the fixed axe, they will be elliptical in the case of the spheroid, and, instead of twice the *square* of the middle diameter, we shall have twice the *product* or *rectangle* of the middle transverse and conjugate; and instead of the square of the end diameter, the rectangle of the end transverse and conjugate; so that the formulæ will be

$$\frac{L(2.T\,C + t\,c)}{1059.108}\ ;\ .0009442\ L(2.T\,C + t\,c)$$

$$\frac{L(2.Q^2 + q^2)}{32.54^2}$$

FORMULÆ FOR GALLONS.

$$\frac{l(2.r^2 + D^2)}{529.554} \; ; \; .0018884\, l(2.r^2 + D^2)$$

$$\frac{l(2.r^2 + D^2)}{23^2}$$

EXAMPLES.

417. A cask in the form of the middle frustum of a prolate spheroid is 40 inches long, the head radius is 12, the bung diameter 32 inches: required the content.

$$\begin{array}{r|l}
\overline{12} & ^2 = 144 \\
\cdot & = 144 \\
32 & = 1024 \\
\hline
 & \; 1312
\end{array}$$

$1312 \times 40 \div 529.554 = 99.1$, as before.

$1312 \times 40 \times .0018884 = 99.1$

By the slide-rule—

$$\frac{40}{23^2}\left\{\begin{array}{r|l}
\overline{12} & ^2 = 10.8 \\
\cdot & = 10.8 \\
32 & = 77.5
\end{array}\right. \qquad \begin{array}{r} \\ \hline 99.1 \end{array}$$

$$\begin{array}{ccccccc} \text{D} & & \text{C} & & \text{D} & & \text{C} \\ 23 & : & 40 & :: & 12 & : & 10.8 \\ & & & & & : & 10.8 \\ & & & :: & 32 & : & 77.5 \\ \hline & & & & & & 99.1 \\ \hline \end{array}$$

418. A cask in the form of the middle frustum of a prolate spheroid is 40 inches long, the bung diameter is 36 inches, the radius of the head 13.5: required the content in gallons.—*Ans.* 125.4.

Problem 25.

419. To find the content of the middle frustum of a sphere, without taking the end diameter.

Rule 1st.—From the square of the middle diameter subtract $\frac{1}{3}$ of the square of the height, multiply the difference by the height, and use the divisors, factors, and gauge points, under the word cylinder.

Formulæ for Gallons.

$$\frac{h(D^2 - \frac{1}{3} \text{ of } . h^2)}{353.036}\ ;\ .00283257\ h(D^2 - \tfrac{1}{3} \text{ of } . h^2)$$

$$\frac{h(D^2 - \frac{1}{3} \text{ of } . h^2)}{18.78^2}$$

Examples.

420. Required the content of the middle zone of a sphere, the axis of which is 44 inches, and the height of the zone 14 inches.

$$\left.\begin{array}{l}\overline{44}\\14\end{array}\right|^{2}\begin{array}{l}=\\=196\ ;\ \tfrac{1}{3}=\end{array}\begin{array}{r}1936\\65.\dot{3}\\\hline 1870.\dot{6}\\\hline\end{array}$$

$1870.\dot{6} \times 14 \div 353.036 = 74.18$ gallons.

$1870.\dot{6} \times 14 \times .00283257 = 74.18$

By the slide-rule—

$$\frac{14}{18.78^{2}}\left\{\left.\begin{array}{l}\overline{44}\\14\end{array}\right|^{2}\right.\begin{array}{l}=\\=7.8\ ;\ \tfrac{1}{3}=\end{array}\begin{array}{r}76.8\\2.6\\\hline 74.4\\\hline\end{array}$$

$$\begin{array}{ccccccccc}\text{D} & & \text{C} & & \text{D} & & & & \text{C}\\18.78 & : & 14 & :: & 44 & : & & & 76.8\\ & & & & & & \text{C} & & \\ & & & & 14 & : & 7.8\ ;\ \tfrac{1}{3} & = & 2.6\\\hline & & & & & & & & 74.4\\\hline\end{array}$$

421. Required the content of the middle zone of a sphere, the axis of which is 22 inches, and the height of the zone 10 inches.—*Ans.* 12.76 gallons.

422. Rule 2nd.—To the square of the end diameter add $\frac{2}{3}$ of the square of the height, multiply the difference by the height, and use the divisors, factors, and gauge points, under the word cylinder.

FORMULÆ FOR GALLONS.

$$\frac{h(d^2 + \tfrac{2}{3} \text{ of } . h^2)}{353.036} \; ; .00283257\, h(d^2 + \tfrac{2}{3} \text{ of } . h^2)$$

$$\frac{h(d^2 + \tfrac{2}{3} \text{ of } . h^2)}{18.78^2}$$

EXAMPLES.

423. Required the content of the middle zone of a sphere, the end diameter being 41.714 inches, and the height 14 inches.

$$\left.\begin{array}{l} \overline{41.714} \\ 14. \end{array}\right|^2 \begin{array}{l} = \\ = 196 \; ; \; \tfrac{2}{3} = \end{array} \begin{array}{r} 1740 \\ 130.\dot{6} \\ \hline 1870.\dot{6} \\ \hline \end{array}$$

$1870.\dot{6} \times 14 \div 353.036 = 74.18$, as before.

$1870.\dot{6} \times 14 \times .00283257 = 74.18$

By the slide-rule—

$$\frac{14}{18.78^2}\left\{ \begin{array}{l} \overline{41.72} \\ 14. \end{array}\right|^2 \begin{array}{l} = \\ = 7.8 \; ; \; \tfrac{2}{3} = \end{array} \begin{array}{r} 69. \\ 5.2 \\ \hline 74.2 \\ \hline \end{array}$$

$$\begin{array}{ccccc} \text{D} & & \text{C} & & \text{D} & & & \text{C} \\ 18.78 & : & 14 & :: & 41.72 & : & & 69. \\ & & & & & & \text{C} & \\ & & & & 14. & : & 7.8 \; ; \; \tfrac{2}{3} = & 5.2 \\ \cline{8-8} & & & & & & & 74.2 \\ \cline{8-8} \end{array}$$

424. What is the content of the middle zone of a sphere, the end diameter of which is 60 inches, and the height of the zone 52 inches?—*Ans.* 795.778 gallons.

Problem 26.

425. To find the content of any frustum of a sphere.

RULE.—Add together the square of the top diameter, the square of the bottom diameter, and $1\frac{1}{3}$ the square of the height, multiply the sum by the height, and use the divisors, factors, and gauge points, under the word paraboloid.

FORMULÆ FOR GALLONS.

$$\frac{h(d^2 + D^2 + 1\tfrac{1}{3}\text{ of}\,.\,h^2)}{706.072}$$

$$.00141628\,h(d^2 + D^2 + 1\tfrac{1}{3}\text{ of}\,.\,h^2)$$

$$\frac{h(d^2 + D^2 + 1\tfrac{1}{3}\text{ of}\,.\,h^2)}{26.57^2}$$

EXAMPLES.

426. Required the content, in gallons, of the shoulder of a still in the form of the frustum of a sphere, the top and bottom diameters being 42 and 36 inches, and the height 30 inches.

$$\begin{array}{rl} \left.\begin{array}{r} \overline{36} \\ 42 \\ 30 \end{array}\right]^2 & \begin{array}{l} = 1296 \\ = 1764 \\ = 900 \end{array} \\ + \frac{1}{3} & = 300 \\ \hline & \ 4260 \end{array}$$

$$4260 \times 30 \div 706.072 = 181 \text{ gallons.}$$

$$4260 \times 30 \times .00141628 = 181.$$

By the slide-rule—

$$\begin{array}{rl} \frac{30}{26.57^2}\left\{ \left.\begin{array}{r} \overline{36} \\ 42 \\ 30 \end{array}\right]^2 \right. & \begin{array}{l} = 55 \\ = 75 \\ = 38.3 \end{array} \\ + \frac{1}{3} & = 12.7 \\ \hline & \ 181. \end{array}$$

$$\begin{array}{ccccccc} \text{D} & & \text{C} & & \text{D} & & \text{C} \\ 26.57 & : & 30 & :: & 36 & : & 55 \\ & & & :: & 42 & : & 75 \\ & & & :: & 30 & : & 38.3 \\ & & & & \frac{1}{3} & = & 12.7 \\ \hline & & & & & & 181. \end{array}$$

427. What is the content, in gallons, of a vessel in the form of the frustum of a sphere, the greater diameter being 52 inches, the less 30, and the depth 18 inches?—*Ans.* 102.88 gallons.

Problem 27.

428. To find the content of the segment of a sphere.

RULE 1ST.—Add together the square of the depth and three times the square of the radius of the top, multiply the sum by the depth, and use the divisors, factors, and gauge points, standing under the word sphere or spheroid.

FORMULÆ FOR GALLONS.

$$\frac{h(h^2 + 3.R^2)}{529.554} \; ; \; .0018884 \cdot h(h^2 + 3.R^2)$$

$$\frac{h(h^2 + 3.R^2)}{23^2}$$

EXAMPLES.

429. A copper basin in the form of the segment of a sphere is 18 inches deep, the radius at the top is 20 inches: required its content in gallons.

$$\begin{array}{rl} \left.\begin{array}{r} \overline{18} \\ 20 \end{array}\right|^2 & \begin{array}{l} = 324 \\ = 400 \end{array} \\ + \text{ twice ditto} & = 800 \\ \hline & \,1524 \end{array}$$

$1524 \times 18 \div 529.554 = 51.8$ gallons.

$1524 \times 18 \times .0018884 = 51.8$

By the slide-rule—

$$\frac{18}{23^2}\left\{\begin{array}{c}\overline{18}\\ 20\\ ..\end{array}\right|^2 \begin{array}{l}= 11.\\ = 13.6\\ = 27.2\\ \hline \quad 51.8\\ \hline\end{array}$$

$$\begin{array}{ccccccc} D & & C & & D & & C\\ 23 & : & 18 & :: & 18 & : & 11\\ & & & :: & 20 & : & 13.6\\ & & & & & & 27.2\\ \cline{7-7} & & & & & & 51.8\\ \cline{7-7}\end{array}$$

430. Required the content of a spherical segment, the radius of the top being 18 inches, and the depth 15 inches.—*Ans.* 33.9 gallons.

431. **Rule 2nd.**—Add together the square of twice the depth and three times the square of the diameter at the top, multiply the sum by the depth, and use the divisors, factors, and gauge points, standing under the words round frustum.

Formulæ for Gallons.

$$\frac{h(\overline{2\,h})^2 + 3.D^2)}{2118.216}$$

$$.0004721\ h(\overline{2\,h})^2 + 3.D^2)$$

$$\frac{h(\overline{2\,h})^2 + 3.D^2)}{46^2}$$

EXAMPLE.

432. A copper basin in the form of the segment of a sphere is 18 inches deep, the diameter at the top is 40 inches: required its content in gallons.

$$\begin{array}{rl}
\left.\begin{array}{r}\overline{36}\\ 40\end{array}\right]^{2} & \begin{array}{l}= 1296\\ = 1600\end{array}\\
+\text{ twice ditto} & = 3200\\
\cline{2-2}
 & \ \ 6096\\
\cline{2-2}
\end{array}$$

$6096 \times 18 \div 2118.216 = 51.8$, as before.
$6096 \times 18 \times .0004721 = 51.8$

By the slide-rule—

$$\frac{18}{46^2}\left\{\left.\begin{array}{c}\overline{36}\\ 40\\ \cdot\cdot\end{array}\right]^{2}\right. \begin{array}{l}= 11.\\ = 13.6\\ = 27.2\\ \hline \ \ 51.8\\ \hline\end{array}$$

$$\begin{array}{ccccccc}
\text{D} & & \text{C} & & \text{D} & & \text{C}\\
46 & : & 18 & :: & 36 & : & 11\\
 & & & & 40 & : & 13.6\\
 & & & & & & 27.2\\
\cline{7-7}
 & & & & & & 51.8\\
\cline{7-7}
\end{array}$$

Problem 28.

433. To find the content of the segment of a spheroid.

CASE 1.—When the base is perpendicular to the fixed axe. Divide the square of the revolving axe by the square of the fixed axe, and multiply the quotient by the difference between three times the fixed axe and twice the height of the segment; multiply the product thus found by the square of the height of the segment, and use the divisors and factors under the word spheroid.

FORMULÆ FOR GALLONS.

$$\frac{v^2(3\,f - 2\,h)h^2}{529.554\,f^2}\ ;\ \frac{v^2}{f^2}.(3\,f - 2\,h)h^2\ .0018884$$

EXAMPLES.

434. The transverse axis of a prolate spheroid is 100 inches, the conjugate 60, and the height of the segment 10: required the solidity.

$$\frac{60^2(300 - 20)10^2}{100^2}\left\{\begin{array}{l}\div\ 529.554\\ \times\ .0018884\end{array}\right\} = 19.035 \text{ gallons.}$$

435. The diameters of an oblate spheroid are 100 and 60: required the content of that segment whose height is 12, and its base perpendicular to the conjugate axe.

$$\frac{100^2(180 - 24)12^2}{60^2}\left\{\begin{array}{l}\div\ 529.554\\ \times\ .0018884\end{array}\right\} = 117.8 \text{ gallons.}$$

436. CASE 2.—When the base is parallel to the fixed axe. Divide the fixed axe by the revolving axe, and multiply the quotient by the difference between three times the revolving axe and twice the height of the segment; multiply the product thus found by the square of the height of the segment, and use the divisors and factors under the word spheroid.

FORMULÆ FOR GALLONS.

$$\frac{f(3\,v - 2\,h)h^2}{529.554\,v}\ ;\ \frac{f}{v}.(3\,v - 2\,h)h^2\ .0018884$$

EXAMPLES.

437. In a prolate spheroid the transverse axe is 100, the conjugate 60, and the height of the segment 12: required the capacity in gallons.

$$\frac{100(180 - 24)12^2}{60}\left\{\begin{matrix} \div\ 529.554 \\ \times\ .0018884 \end{matrix}\right\} = 70.7 \text{ gallons.}$$

438. Required the content of the segment of a prolate spheroid, its height being 6 inches, and the axes 50 and 30.

$$\frac{50(90 - 12)6^2}{30}\left\{\begin{matrix} \div\ 529.554 \\ \times\ .0018884 \end{matrix}\right\} = 8.83 \text{ gallons.}$$

UNGULAS OR HOOFS.

439. An ungula is any part of a solid cut off by a plane not parallel to the base. Liquids contained in vessels placed in an inclined position necessarily assume the form of ungulas.

440. To find the content of the ungula of a cylinder when the section passes obliquely through the opposite sides. Take half the sum of the greatest and least depths for a mean depth, multiply by the square of the diameter of the cylinder, and use the divisor or factor for cylinder.

441. To find the content of a cylindrical ungula when its base is less than a semi-circle. Multiply the area of the base of the ungula by the difference between half the diameter of the vessel and the versed sine of the base of the ungula. Subtract the product thus obtained from $\frac{1}{12}$ of the cube of the chord of the base, multiply the remainder by the height of the ungula, and divide the product by the versed sine of the base, and use the divisor or factor under square prism.

442. When the base is a semi-circle. Multiply the square of the diameter of the vessel by the height of the ungula, and use the divisor or factor under square frustum.

443. When the base is greater than a semi-circle. Multiply the area of the base by the difference between the versed sine of the base of the ungula and half the diameter of the vessel. To the product thus obtained add $\frac{1}{12}$ of the cube of the chord of the base, multiply the sum by the height of the ungula, divide by the versed sine of the base, and use the divisor or factor under square prism.

444. The above rules for cylinders, or innumerilateral prisms, are applicable to all prisms.

445. Liquor in an inclined frustum of a pyramid will assume the form of a wedge, a prismoid, or both, and is to be gauged accordingly.

446. To find the content of a conical ungula when it stands upon its greater base, and in such position that the liquor just covers the whole of the bottom. Multiply the product of the top and bottom diameters by the mean proportional between them, subtract the last product from the cube of the bottom diameter, divide the remainder by the difference of the diameters, multiply the quotient by the perpendicular height of the ungula, and use the divisor or factor under the word cone.

447. When it stands upon its less base in a similar manner. Multiply the product of the two diameters by the mean proportional between them, from the product thus obtained subtract the cube of the bottom diameter, divide the remainder by the difference of

the diameters, multiply the quotient by the perpendicular height of the ungula, and use the divisor or factor under the word cone.

448. When the section intersects opposite sides of the frustum, it forms the frustum of a cone and an ungula; the contents are, therefore, to be found separately, and added together.

Problem 28.

449. To find the content of a circular vessel with irregularly protuberant sides, by a competent odd number of equidistant diameters. This is effected by taking the solid in detail, in successive strata, as we formerly took the area in detail, in successive strips.

450. The general rule for frustums is to add together the square of the top diameter, the square of the bottom diameter, and four times the square of the intermediate diameter, and to multiply by one-sixth of the length—that is, by one-third of the distance between the diameters. Hence, by taking the solid in successive portions, bounded and bisected by equidistant planes, we shall have the square of the first diameter, the square of the third diameter, and four times the square of the second; then the square of the third, the square of the fifth, and four times the square of the fourth; and so on. Consequently, as the segments are taken in pairs, there must always be

an even number of them, contained, of course, between an odd number of planes; and we shall have the square of the first and last diameters, twice the square of all the other odd diameters, and four times the square of all the even diameters, multiplied by one-third of the common distance: and so they run on, as before,

$$\frac{L(1\,a^2 + 4\,b^2 + 1\,c^2)}{1 + 4 + 1};$$

$$\frac{L(1\,a^2 + 4\,b^2 + 2\,c^2 + 4\,d^2 + 1\,e^2)}{1 + 4 + 2 + 4 + 1};$$

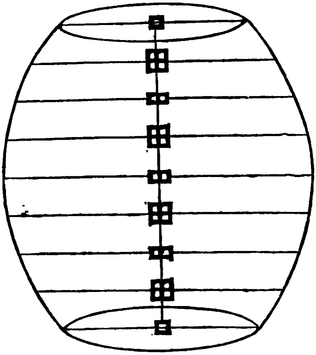
$$\frac{L(1\,a^2 + 4\,b^2 + 2\,c^2 + 4\,d^2 + 2\,e^2 + 4\,f^2 + 1\,g^2)}{1 + 4 + 2 + 4 + 2 + 4 + 1}.$$

Hence the rule for obtaining the content of an irregular circular vessel is as follows:—

451. **Rule.**—Place in a line the figures $\underline{4}$, $\underline{2}$, $\underline{1}$, and draw a mark under each. Under the 4 place the square of all the even diameters, under the 2 the square of all the odd diameters, except the first and last, and under the 1 the square of the first and last diameters; add up the three columns separately, and multiply each column by the figure standing over it; add up the three products so obtained, multiply the sum by the common distance, and use the divisors, factors, and gauge points, standing under the word cone.

EXAMPLES.

452. What is the content, in gallons, of a large circular vessel whose depth is 96 inches, and the diameters of which, commencing with the bottom, and proceeding upwards, at equal distances of 12 inches each, are $\underset{\prime}{80}$; $\underset{\prime\prime\prime\prime}{83}$; $\underset{\prime\prime}{86}$; $\underset{\prime\prime\prime\prime}{88}$; $\underset{\prime\prime}{90}$; $\underset{\prime\prime\prime\prime}{89}$; $\underset{\prime\prime}{87}$; $\underset{\prime\prime\prime\prime}{84}$; and $\underset{\prime}{81}$ inches?



	4		2		1
83^2	= 6889	86^2	= 7396	80^2	= 6400
88^2	= 7744	90^2	= 8100	81^2	= 6561
89^2	= 7921	87^2	= 7569		12961
84^2	= 7056		23065		
	29610		46130		
	118440				
	46130				
	12961				

$$177531 \times 12 \left\{\begin{matrix} \div 1059.108 \\ \times .0009442 \end{matrix}\right\} = 2011.5 \text{ gallons.}$$

By the slide-rule—

$$\frac{12}{32.54^2}\left\{\begin{array}{llllll}
 & \underline{\;4\;} & & \underline{\;2\;} & & \underline{\;1\;}\\
\left.\begin{array}{r}83\\88\\89\\84\end{array}\right|^2 & \begin{array}{r}=78\\=87.7\\=89.8\\=\underline{80.}\\\underline{335.5}\\1342.\end{array} & \left.\begin{array}{r}86\\90\\87\end{array}\right|^2 & \begin{array}{r}=83.8\\=91.8\\=\underline{85.8}\\\underline{261.4}\\522.8\end{array} & \left.\begin{array}{r}80\\81\end{array}\right|^2 & \begin{array}{r}=72.5\\=\underline{74.2}\\146.7\end{array}
\end{array}\right.$$

$$\begin{array}{r}
1342.\\
522.8\\
\underline{146.7}\\
\underline{\underline{2011.5}}\text{ gallons.}
\end{array}$$

453. Required the content, in gallons, of a circular vessel, whose depth is 120 inches, and the diameters of which, commencing with the top, and proceeding downwards, at equal distances of 20 inches each, are 124 ; 146 ; 161 ; 164 ; 166 ; 157 ; and 144 inches.
′ ′′′′ ′′ ′′′′ ′′ ′′′′ ′

$$\begin{array}{llllll}
 & \underline{\;4\;} & & \underline{\;2\;} & & \underline{\;1\;}\\
\left.\begin{array}{r}146\\164\\157\end{array}\right|^2 & \begin{array}{r}=21316\\=26896\\=\underline{24649}\\\underline{72861}\\291444\end{array} & \left.\begin{array}{r}161\\166\end{array}\right|^2 & \begin{array}{r}=25921\\=\underline{27556}\\\underline{53477}\\106954\end{array} & \left.\begin{array}{r}124\\144\end{array}\right|^2 & \begin{array}{r}=15376\\=\underline{20736}\\36112\end{array}
\end{array}$$

$$\begin{array}{r}
291444\\
106954\\
\underline{36112}\\
434510
\end{array}\times 20\left\{\begin{array}{l}\div 1059.108\\ \\ \times .0009442\end{array}\right\}=\begin{array}{l}8205.28\\\text{gallons.}\end{array}$$

454. Required the content, in gallons, of a circular

vessel, the diameters of which are $\underset{/}{108.4}$; $\underset{////}{123.6}$; $\underset{//}{130.8}$; $\underset{////}{136.6}$; $\underset{//}{136.2}$; $\underset{////}{131.8}$; $\underset{//}{124.2}$; $\underset{////}{114.8}$; and $\underset{/}{96.8}$ inches, and the common distance of the diameters 18.3 inches.—*Ans.* 6587.34 gallons.

455. To find the content of an elliptical vessel, or vessel with unequal cross diameters and irregularly protuberant sides, by a competent number of equidistant diameters.

456. This is the same rule as the preceding, except that, as the diameters are unequal, their rectangle is no longer their square; we have, therefore, the rectangles of the first and last cross diameters, twice the rectangles of all the other odd cross diameters, and four times the rectangles of all the even cross diameters, multiplied by one-third of the common distance. Hence the rule for obtaining the content of an irregular elliptical vessel is as follows:—

457. Rule.—Place in a line the figures $\underline{4}$, $\underline{2}$, $\underline{1}$, and draw a mark under each. Under the 4 place the products of all the even cross diameters, under the 2 the products of all the odd cross diameters, except the first and last, and under the 1 the products of the first and last cross diameters; add up the three columns separately, and multiply each column by the figure standing over it; add up the three products so obtained, multiply the sum by the common distance, and use the divisors and factors under the word cone.

EXAMPLES.

458. Required the content, in gallons, of a vessel slightly elliptical, whose depth is 96 inches, and the diameters of which, commencing with the bottom pair, and proceeding upwards, at equal distances of 12 inches each, are $78 \underset{/}{\times} 82$; $81 \underset{////}{\times} 85$; $84 \underset{//}{\times} 88$; $86 \underset{////}{\times} 90$; $88 \underset{//}{\times} 92$; $87 \underset{////}{\times} 91$; $85 \underset{//}{\times} 89$; $82 \underset{////}{\times} 86$; and $79 \underset{/}{\times} 83$ inches.

$$
\begin{array}{rrr}
4 & 2 & 1 \\
81 \times 85 = \overline{6885} & 84 \times 88 = \overline{7392} & 78 \times 82 = \overline{6396} \\
86 \times 90 = 7740 & 88 \times 92 = 8096 & 79 \times 83 = 6557 \\
87 \times 91 = 7917 & 85 \times 89 = 7565 & \overline{12953} \\
82 \times 86 = 7052 & \overline{23053} & \\
\overline{29594} & \overline{46106} & \\
\overline{118376} & & \\
46106 & & \\
12953 & & \\
\overline{177435} \times 12 \left\{ \begin{array}{l} \div\ 1059.108 \\ \times\ .0009442 \end{array} \right\} & = 2010.4 \text{ gallons.} &
\end{array}
$$

459. This vessel is similar to the one in question, 452, from which it differs only in this, that of each pair of cross diameters, one is 2 inches less, and the other 2 inches more, than the corresponding one of the circular vessel, and is introduced for the purpose of showing the error of computing the content by mean diameters; for, as the circle, as before observed,

contains the greatest area within the shortest bounding line, the taking a mean diameter always gives the content in excess, the arithmetical mean being greater than the geometrical mean or mean proportional.

460. For, let the smaller diameter be represented by A, and the greater by $A + n$; then $A + \frac{n}{2}$ is the arithmetical mean; the square of which, $A^2 + An + \frac{n^2}{4}$, representing the circle; $(A + n) A = A^2 + An$, will represent the ellipse; hence the circle upon the mean diameter exceeds the ellipse by one-fourth of the square of the difference of the axes of the latter, multiplied by the circular factor.

461. To illustrate this numerically, suppose the diameters of the ellipse to be 7 and 5; then $7 \times 5 = 35$, represents the ellipse; and the square of 6, (the mean between 7 and 5,) $= 36$, represents the circle, exceeding the other by 1, which is one-fourth of the square of the difference between 7 and 5; and $1 \times .00283257 = .00283257$, the difference of the areas.

462. It must be observed, however, that even this mode of computing the content by the rectangles of cross diameters, though more correct than by mean diameters, will not be *perfectly* correct unless the sections are *truly* elliptical. If the shape, however,

of the vessel deviates but little from such figure, the capacity so found will be sufficiently near for all practical purposes. When the vessel is of a very irregular oval form, and also protuberant in the middle, the following rule must be used:—

463. To find the content of a vessel irregularly oval, with protuberant sides.

RULE.—Take the transverse diameter at the top, bottom, and middle of the vessel, crossed by an adequate odd number of equidistant ordinates; then, by the rule given in 294, find thrice the top area, thrice the bottom area, and four times thrice the middle area; their sum will be 18 mean areas. Multiply this by the depth of the vessel, and it will give 18 mean capacities; then, for a divisor, take 18 times the square prism divisor, and for a factor its reciprocal, viz.:—

For gallons—

Divisor	4990.932
Factor	.000200363

For bushels—

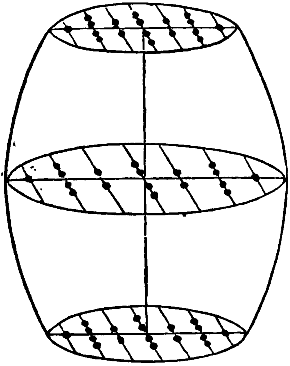
Divisor	39927.456
Factor	.000025045

For cubic feet—

Divisor	31104
Factor	.00003215

EXAMPLE.

464. Required the content of an irregularly oval and protuberant vessel 40 inches deep, with the following dimensions:—



	Common Distance	Double Ordinates, or Cross Parallels.							Height of Terminal Segment.
Top ...	8	22	34	41	43	40	35	23	4
Middle	10	30	43	49	52	48	44	31	5
Bottom	9	27	39	46	48	45	40	28	4.5
		•	••••	••	••••	••	••••	•	

For the top areas—

4	2	1
34	41	22
43	40	23
35	81	45
112	162	8
448		360
162		
45		
655		
8		
5240		
360		
5600 = thrice the top area.		

For the middle areas—

4	2	1
43	49	30
52	48	31
44	97	61
139	194	10
556		610
194		
61		
811		
10		
8110		
610		
8720 = thrice the middle area.		
4		
34880 = twelve times do.		

For the bottom areas—

$$\begin{array}{rlrr}
4 & & 2 & 1 \\
\overline{39} & & \overline{46} & \overline{27} \\
48 & & \overline{45} & 28 \\
40 & & 91 & \overline{55} \\
\overline{127} & & \overline{182} & 9 \\
\overline{508} & & & \overline{495} \\
182 & & & \\
55 & & & \\
\overline{745} & & & \\
9 & & & \\
\overline{6705} & & & \\
495 & & & \\
\overline{7200} & = \text{thrice the bottom area.} & & \\
5600 & & & \\
34880 & & & \\
\overline{47680} & = 18 \text{ mean areas.} & & \\
40 & \text{depth.} & & \\
\underline{\overline{1907200}} & = 18 \text{ capacities.} & &
\end{array}$$

$$1907200 \left\{ \begin{array}{l} \div\ 4990.932 \\ \\ \times\ .000200363 \end{array} \right\} = 382.13 \text{ gallons.}$$

465. Having found thrice the top and bottom areas, we might have divided by 3 to obtain the area; and having found thrice the middle area, we might have added to it one-third of itself to obtain 4 times the middle area, and then have selected the square frustum divisor and factor. As the number 3, however, would, in each instance, probably, give a recurring

decimal and a needless trouble, it is best avoided by the mode recommended.

466. To find the content of a vessel irregularly oval, with straight sides, but unequal top and bottom diameters—that is, in the shape of the frustum of an irregularly elliptical cone.

Here we have to take the transverse diameters of the top and bottom only, crossed by an adequate odd number of equidistant ordinates; their sum, agreeably to the rules already laid down, will give the dimensions for obtaining four times thrice the middle area.

Example.

467. Required the content of an irregularly oval, straight-sided vessel, 40 inches deep, with the following dimensions:—

	Common Distance	Double Ordinates, or Cross Parallels.							Height of Terminal Segment.
Top ...	8	22	34	41	43	40	35	23	4
Bottom	10	30	43	49	52	48	44	31	5
Sum ...	18	52	77	90	95	88	79	54	9
		.	....	..	....	..	....	.	

For the top areas—

4	2	1
34	41	22
43	40	23
35	81	45
112	162	8
448		360
162		
45		
655		
8		
5240		
360		
5600 = thrice the top area.		

For the bottom areas—

4	2	1
43	49	30
52	48	31
44	97	61
139	194	10
556		610
194		
61		
811		
10		
8110		
610		
8720 = thrice the bottom area.		

For the middle areas—

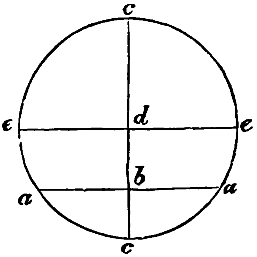
4	2	1
77	90	52
95	88	54
79	178	106
251	356	18
1004		848
356		106
106		1908
1466		
18		
11728		
1466		
26388		
1908		
28296 = 12 times the middle area.		
5600		
8720		
42616 = 18 areas.		
40		
1704640 = 18 capacities.		

$$1704640\left\{\begin{matrix}\div\ 4990.932\\ \\ \times\ .000200363\end{matrix}\right\} = 341.546 \text{ gallons.}$$

468. If a vessel should be very irregular, in some parts increasing, and in others decreasing, in breadth, alternately, then diameters should be taken, one at the commencement and one at the end of each irregular piece, and each of these pieces should be

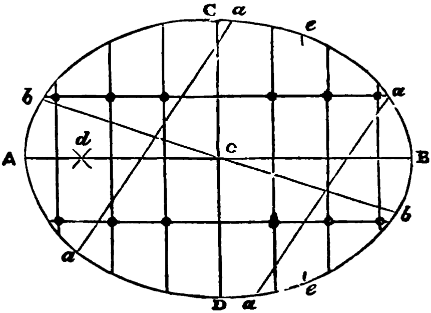
taken by itself, and its capacity computed by bisecting it by an intermediate ordinate of its own.

469. To quarter a circle—



Draw any line *a a*, bisect it in *b*, and through *b* draw *c c* at right angles to *a a*. Bisect *c c* in *d*, and through *d* draw *e e* parallel to *a a*. The points *c*, *e*, *c*, *e*, divide the circumference into 4 equal parts.

470. To quarter an ellipse, and cross it with equidistant ordinates—



Draw any two parallel lines *a a*, *a a*. Bisect each of them, and through the centres draw *b b*. Bisect *b b* in *c*; *c* is the centre of the ellipse. From *c*, with any convenient radius, touch the periphery in *e* and *e*. From *e*, *e*, with a greater radius, draw intersecting arcs at *d*. Through *d* and *c* draw the transverse A B, and at right angles to A B, through *c*, draw the conjugate C D. On each side of A B draw a parallel, and along these parallels, on each side of C D, set off any convenient distance, as indicated by the dots, and through the dots draw the ordinates.

SUMMARY OF FORMULÆ FOR SOLIDS.

1. Square prism $= \dfrac{h\,s^2}{\text{square prism}}$
2. Rectangular prism $= \dfrac{l\,b\,d}{\text{square prism}}$
3. Parabolic prism $= \dfrac{d\,b\,h}{\text{parabolic prism}}$
4. Polygonal prism $= \dfrac{h\,s^2}{\text{polygonal prism}}$
5. Irregular prism $= h \times$ area of base
6. Cylinder $= \dfrac{h\,d^2}{\text{cylinder}}$
7. Elliptic cylinder $= \dfrac{h\,t\,c}{\text{cylinder}}$

8. Square pyramid $= \dfrac{h\,s^2}{\text{square pyramid}}$

9. Rectangular pyramid $= \dfrac{l\,b\,d}{\text{square pyramid}}$

10. Polygonal pyramid $= \dfrac{h\,s^2}{\text{polygonal pyramid}}$

11. Cone $= \dfrac{h\,d^2}{\text{cone}}$

12. Elliptic cone $= \dfrac{h\,t\,c}{\text{cone}}$

13. Sphere or spheroid $= \dfrac{f\,v^2}{\text{sphere}}$

14. Paraboloid $= \dfrac{h\,d^2}{\text{paraboloid}}$

15. Parabolic spindle $= \dfrac{l\,d^2}{\text{parabolic spindle}}$

16. Hyperboloid $= \dfrac{h(\mathrm{D}^2 + \overline{2\,\delta}^2)}{\text{round frustum}}$

Or, $\dfrac{h(\mathrm{R}^2 + \delta^2)}{\text{sphere}}$

17. Spindles in general $= \dfrac{l(\mathrm{D} + \overline{2\,\delta}^2)}{\text{round frustum}}$

Or, $\dfrac{l(\mathrm{R}^2 + \delta^2)}{\text{sphere}}$

18. Frustum of pyramid $= \dfrac{l(s^2 + \mathrm{S}^2 + \overline{s + \mathrm{S}}^2)}{\text{polygonal frustum}}$

19. Frustum of cone $= \dfrac{l(d^2 + D^2 + \overline{d + D}|^2)}{\text{round frustum}}$

Or, $\dfrac{l(r^2 + R^2 + \overline{r + R}|^2)}{\text{sphere}}$

20. Frustum of paraboloid $= \dfrac{h(d^2 + D^2)}{\text{paraboloid}}$

21. Frustum of hyperboloid $= \dfrac{h(d^2 + D^2 + \overline{2\ \delta}|^2)}{\text{round frustum}}$

Or, $\dfrac{h(r^2 + R^2 + \delta^2)}{\text{sphere}}$

22. Middle frustum of parabolic spindle

$$= \frac{l(2.D^2 + d^2 - \frac{1}{10} \text{ of } . \overline{2 \text{ diff.}}|^2)}{\text{cone}}$$

Or, $\dfrac{l(D^2 + 2.r^2 - \frac{1}{5} \text{ of } . \overline{D - d}|^2)}{\text{sphere}}$

23. Middle frustum of spindles in general

$$= \frac{l(d^2 + D^2 + \overline{2\ \delta}|^2)}{\text{round frustum}}$$

Or, $\dfrac{l(r^2 + R^2 + \delta^2)}{\text{sphere}}$

24. Middle frustum of sphere or spheroid, with ends perpendicular to fixed axe—

$$\frac{l(2.D^2 + d^2)}{\text{cone}}; \text{ or } \frac{l(2.r^2 + D^2)}{\text{sphere}}$$

With ends parallel to fixed axe—

$$\frac{L(2.T\ C + t\ c)}{\text{cone}}$$

25. Middle frustum of sphere

$$= \frac{h(\mathrm{D}^2 - \frac{1}{3} \text{ of } . h^2)}{\text{cylinder}}; \text{ or, } \frac{h(d^2 + \frac{2}{3} \text{ of } . h^2)}{\text{cylinder}}$$

26. Any frustum of sphere

$$= \frac{h(d^2 + \mathrm{D}^2 + 1\frac{1}{3} \text{ of } . h^2)}{\text{paraboloid}}$$

27. Segment of sphere $= \dfrac{h(h^2 + 3.\mathrm{R}^2)}{\text{sphere}}$

$$\text{Or, } \frac{h(\overline{2\,h})^2 + 3.\mathrm{D}^2)}{\text{round frustum}}$$

28. Segment of spheroid, with base perpendicular to fixed axe $= \dfrac{v^2}{f^2} . \dfrac{(3\,f - 2\,h)h^2}{\text{spheroid}}$

With base parallel to fixed axe $= \dfrac{f}{v} . \dfrac{(3\,v - 2\,h)h^2}{\text{spheroid}}$

471. It has already been remarked, that in the formulæ for incomplete figures the denominator is equal to the sum of the coefficients of the numerator, when the terms of such numerator represent the variable breadths; in planes, ordinates; in solids, sides or diameters. Therefore, however much the formulæ for solids appear to vary in detail, they all come under the one comprehensive form of

$$\frac{\mathrm{L}(x\,\mathrm{S}^2 + y\,\mathrm{S}\,s + z\,s^2)}{(x + y + z)k}$$

if equiangular; and of

$$\frac{\mathrm{L}(x\,\mathrm{D}^2 + y\,\mathrm{D}\,d + z\,d^2)}{(x + y + z)c}$$

if round; where L denotes length, S and s greater and less sides, D and d greater and less diameters, x, y, z, variable coefficients depending on the nature of the solid; k, 277.274, the number of cubic inches in a gallon; and c, 353.036, the number of cylindrical inches in a gallon.

472. When the imperfect figure, or frustum, is completed into the entire solid, and goes off to a point, as in the pyramid, cone, and spindle; or runs round, as in the sphere, spheroid, paraboloid, and hyperboloid; s and d vanish; and when the less end is expanded till it becomes equal to the greater, and the frustum of the pyramid passes into the prism, and that of the cone into the cylinder, s becomes equal to S, and d to D. The formulæ above given comprehend, therefore, the complete solids as well as their imperfect frustums.

473. To begin with the frustum of the square pyramid. The rule for this is the one applicable to all regular solids—viz.,

$$\frac{L(1\ S^2 + 4\ \Sigma^2 + 1\ s^2)}{(1 + 4 + 1)k} = \frac{L(S^2 + \overline{S + s)}^2 + s^2)}{6\ k} =$$

$$\frac{L(2.S^2 + 2\ S\ s + 2\ s^2)}{6\ k} = \frac{L(S^2 + S\ s + s^2)}{3\ k}$$

474. Let the frustum run up into the complete pyramid, then $s = 0$, and its multiples vanish, and $\frac{L(2\ S^2 + 2\ S\ s + 2\ s^2)}{6\ k}$ becomes $\frac{L\ S}{3\ k} = \frac{1}{3}$ the prism.

475. Let the less end of the frustum expand till it passes into the prism, then $s = S$, and the formula becomes $\frac{L(2\,S^2 + 2\,S^2 + 2\,S^2)}{6\,k} = \frac{L\,S^2}{k}$

476. The same obviously applies to the cone and cylinder, D and d simply taking the place of S and s, and c of k.

477. The rule for the middle frustum of the sphere or spheroid is $\frac{L(2.D^2 + d^2)}{3\,c}$

This is also a modification of the general rule for frustums, d and d, the end diameters, being equal and D taking the place of the intermediate diameter, the formula being

$$\frac{L(1\,d^2 + 4\,D^2 + 1\,d^2)}{(1 + 4 + 1)c} = \frac{L(4\,D^2 + 2\,d^2)}{6\,c} = \frac{L(2.D^2 + d^2)}{3\,c}$$

478. Let it run into the complete sphere or spheroid, then d vanishes, and we have

$$\frac{L(4\,D^2)}{6\,c} = \frac{L(2.D^2)}{3\,c} = \frac{2\,L\,D^2}{3\,c} = \tfrac{2}{3} \text{ of the cylinder.}$$

479. Let the less end expand into the cylinder, then $d = D$, and the formula becomes

$$\frac{L(1\,D^2 + 4\,D^2 + 1\,D^2)}{(1 + 4 + 1)c} = \frac{L\,6\,D^2}{6\,c} = \frac{L\,D^2}{c}$$

480. So with the frustum of the hyperboloid, the rule for which is

$$\frac{L(1\,D^2 + 4\,\delta^2 + 1\,d^2)}{(1 + 4 + 1)c} = \frac{L(D^2 + 4\,\delta^2 + d^2)}{6\,c}$$

481. Let the frustum pass into the complete solid, and d^2 vanishes; and we have $\dfrac{L(D^2 + 4\,\delta^2)}{6\,c}$

482. Let the less end of the frustum expand, and it also passes into the cylinder.

483. The formula for the frustum of the parabolic spindle is $\dfrac{L(2.D^2 + d^2 - 2\overline{(D - d)}^2)}{3\,c}$

This, reduced, becomes

$$\frac{L(20\,D^2 + 10\,d^2 - 4\,D^2 + 8\,D\,d - 4\,d^2)}{30\,c}$$

$$= \frac{L(8\,D^2 + 4\,D\,d + 3\,d^2)}{(8 + 4 + 3)c}$$

$$= \frac{L(8\,D^2 + 4\,D\,d + 3\,d^2)}{15\,c}.$$

484. Let the solid be completed, then $d = 0$, and its multiples vanish, and the formula becomes

$$\frac{L(8\,D^2)}{15\,c} = \frac{8\,L\,D^2}{15\,c} = \tfrac{8}{15} \text{ of the cylinder.}$$

485. Let the less end of this frustum expand, and it also passes into the cylinder.

486. The formula for the frustum of the paraboloid

is also a modification of the general rule for frustums.
$\frac{L(1\ D^2 + 4\ \delta^2 + 1\ d^2)}{(1 + 4 + 1)c}$ = (the squares of the equidistant diameters being in arithmetical progression)

$$\frac{L(3\ D^2 + 3\ d^2)}{6\ c} = \frac{L(D^2 + d_2)}{2\ c}$$

487. Let the solid be completed, then d^2 vanishes, and it becomes $\frac{L\ D^2}{2\ c}$ = ½ the cylinder.

488. Let the less end of this frustum expand, and it also passes into the cylinder.

489. It will be evident, that unless the denominator were equal to the sum of the coefficients of the numerator, the formula could not merge into that of the cylinder.

CASK GAUGING.

490. Casks were formerly divided into four varieties, according to the form of the solid to which they were supposed to bear the nearest resemblance. These four varieties were the following:—

1st. The middle frustum of a prolate spheroid.

2nd. The middle frustum of a parabolic spindle.

3rd. The frustums of two paraboloids joined together at the greater end.

4th. The frustums of two cones joined in like manner.

491. The formulæ for these have already been given under the various solids; they are here grouped together for the sake of comparison.

H denotes head diameter, B bung diameter, L length.

1st. Fr. pro. sphd. $\dfrac{L(2.B^2 + H^2)}{32.54^2}$

2nd. Fr. par. spin. $\dfrac{L(2.B^2 + H^2 - \frac{1}{10} \text{ of } \overline{.2 \text{ diff.}}|^2)}{32.54^2}$

3rd. Fr. two parab. $\dfrac{L(B^2 + H^2)}{26.57^2}$

4th. Fr. two co. $\dfrac{L(B^2 + H^2 + \overline{B + H}|^2)}{46^2}$

492. Suppose a cask 40 inches long, the bung diameter 32, the head 24: required its content for the four varieties.

1st. Pro. sphd.

$$\frac{40}{32.54^2}\left\{\begin{array}{c|l} \overline{32}\,^2 & = 38.7 \\ \cdot & = 38.7 \\ 24 & = \underline{21.7} \\ & \ 99.1 \text{ gallons.} \end{array}\right.$$

2nd. Par. spin.

$$\frac{40}{32.54^2}\left\{\begin{array}{r|l} \overline{32}\,^2 & = 38.7 \\ \cdot & = 38.7 \\ \underline{24} & = \underline{21.7} \\ \underline{8} & \ 99.1 \\ \tfrac{1}{10}\ \ 16 & = \underline{.97} \\ & \ 98.13 \text{ gallons.} \end{array}\right.$$

3rd. Two parab. $$\frac{40}{26.57^2}\left\{\begin{array}{c}\overline{32}\\ 24\end{array}\right|^2 \begin{array}{l}= 58.1\\ = 32.6\\ \hline 90.7 \text{ gallons.}\\ \hline\end{array}$$

4th. Two co. $$\frac{40}{46^2}\left\{\begin{array}{c}\overline{32}\\ 24\\ 56\end{array}\right|^2 \begin{array}{l}= 19.4\\ = 10.87\\ = 59.2\\ \hline 89.47 \text{ gallons.}\\ \hline\end{array}$$

493. With respect to these varieties, it may be observed, that the frustum of the prolate spheroid gives the content too great; as, in such figure, the protuberance is gradual from the head to the bung; whereas, in real casks, it will generally be found, that, for some distance from the head either way, they are very nearly straight.

494. The frustum of the parabolic spindle is nearer, but the formula is somewhat tedious, and the deduction, after all, trifling and *insufficient*; its form differs very little from the frustum of the prolate spheroid, and the content will, therefore, still be too great.

495. By laying down a parabola, with the requisite divergence, it will be apparent to the eye in a moment that the curve is too nearly coincident with a straight line; and by joining two together, it will be equally

obvious, that, while they are very nearly, if not perfectly, of the proper form about the head, they meet with too sharp a ridge in the middle, and have not sufficient protuberance about the intermediate diameter. The content of casks computed upon the assumption that they are made up of two such solids will be found deficient.

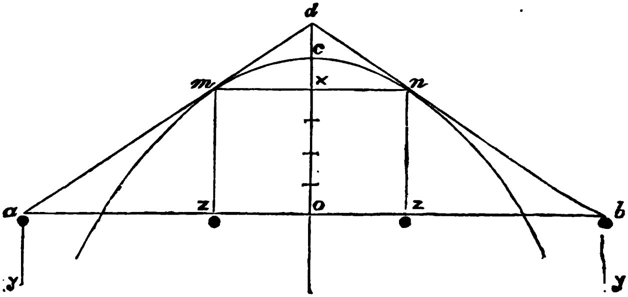
496. As for the frustums of two cones running in a straight line from head to bung, they make a ridiculous kind of barrel, and are not worth consideration. It is indeed difficult to conceive that anyone could have ever seriously proposed so grotesque a resemblance to a cask; and, as the rule for finding the content was usually exhibited under the form $\frac{L(B^2 + B\,H + H^2)}{32.54^2}$, it was less suited for the Slide-rule than the others, requiring the tedious intermediate process of finding a mean proportional between B and H, and therefore two separate settings of the slide. By simply doubling the numerator and denominator, however, it was shown that the content might be computed on the rule by one setting of the slide, with the formula $\frac{L(B^2 + H^2 + \overline{B + H}|^2)}{46^2}$; but this, though easy and correct for such figure, is not suitable for a cask, because no cask is ever made in such a form; nor,

indeed, could be, unless the staves were cut in two separate straight pieces, and subsequently joined together in the middle.

497. To what form, then, can casks be assimilated? From the method employed in making them, it is quite obvious, that if their shape ever does exactly agree with any definite geometrical solid, it must be perfectly fortuitous. The probability is that they combine the spheroid and the paraboloid, or the parabolic spindle and the cone, in varying proportions, protuberant chiefly in the middle, and straight, or very nearly so, towards the ends.

498. On the assumption that casks are a compound of two varieties, Dr. Hutton, at the suggestion, as he states, of Mr. Davidson, formed an imaginary cask, in the following proportions:—

$\frac{1}{3}$ *frustum of cone* + $\frac{1}{3}$ *middle frustum of parabolic spindle* + $\frac{1}{3}$ *frustum of cone*



499. Let *amcnb* represent one of the staves, which is purposely drawn very much arched, the plainer to show the figure. Join *ab*; bisect it in *o*, and trisect it in *z* and *z*; prolong *am* and *bn* to meet in *d*; join *do*, and let *mcn* be the curve of a parabola, whose vertex is *c*, and axis *co*; draw *ay* and *by* (parts of the head diameters), *mz* and *nz*, all parallel to *do*, and join *mn*. Then *ad* is a tangent to the parabola at the point *m*, and $\therefore dc = cx$. Now, by the similarity of the triangles *mxd*, *aod*, *mx* being $\frac{1}{3}$ of *ao*, *xd* is $\frac{1}{3}$ of *od* $\therefore xc = \frac{1}{6}$ of $od = \frac{1}{5}$ of *oc* $= \frac{1}{10}$ of twice $oc = \frac{1}{10}$ of $.\overline{\text{the bung minus the head}}$ $= \dfrac{B - H}{10} \therefore 2xc = \dfrac{B - H}{5}$. Therefore $B - 2xc$ $= B - \dfrac{B - H}{5} = \dfrac{4B + H}{5}$, the end diameter of the parabolic spindular portion, the formula for which is $\dfrac{8D^2 + 4Dd + 3d^2}{15c} \times \dfrac{L}{3}$ (it being $\frac{1}{3}$ of the entire length of the cask, which is L).

Substituting B (the bung diameter) for D, and $\dfrac{4B + H}{5}$ (the end diameter) for *d*, this becomes

$$\frac{328B^2 + 44BH + 3H^2}{25 \times 15 \times 3c = (328 + 44 + 3)3c}.L = \text{the con-}$$

tent of the parabolic spindular portion.

Then, for the two conical frustums, we have

$\frac{D^2 + D\,d + d^2}{3\,c} \times \frac{2\,L}{3}$ (being together $\frac{2}{3}$ of the entire length). In this formula D becomes $\frac{4\,B + H}{5}$ (the diameter at the base of the cone), and $d = H$ (the head diameter): substituting these values, we obtain

$$\frac{160\,B^2 + 280\,B\,H + 310\,H^2}{25 \times 15 \times 3\,c = (160 + 280 + 310)\,\frac{3}{2}\,c}.L = \text{the}$$

content of the two conical frustums.

The sum of the three frustums, therefore, will be $\frac{488\,B^2 + 324\,B\,H + 313\,H^2}{1125\,c}.L =$ the content of the cask.

500. As the coefficient of H^2 is a prime number, the fraction does not admit of being reduced to lower terms; but, dividing each by $12\frac{20}{39}$, we have

$$\frac{L(39\,B^2 + 25\frac{109}{122}\,B\,H + 25\frac{7}{488}\,H^2)}{89\frac{443}{488}\,c} =$$

$$\frac{L(39\,B^2 + 25\frac{109}{122}\,B\,H + 25\frac{7}{488}\,H^2)}{(39 + 25\frac{109}{122} + 25\frac{7}{488})\,c}$$

Then, by a little compensation, increasing the coefficient of the second term, diminishing the third, and adjusting the denominator accordingly, we have

$$\frac{L(39\,B^2 + 26\,B\,H + 25\,H^2)}{90\,c} =$$

$$\frac{L(39\,B^2 + 26\,B\,H + 25\,H^2}{90 \times 353.036} =$$

$$L(39\,B^2 + 26\,B\,H + 25\,H^2)\,.000031473$$

501. This is a laborious formula for what is, after all, only hypothetical; it also makes the staves very straight, as is readily observable; and, as it stands, it is wholly unsuited to the Slide-rule. By preparing a table, however, of divisors, factors, and gauge points, it may be rendered of as easy application as the usual formulæ.

502. Conceive a square pyramid to be circumscribed by a square prism. The content of the prism is $\frac{L\,s^2}{k}$. The pyramid being $\frac{1}{3}$ of this, is $\frac{L\,s^2}{3\,k}$. The pyramid being thus in a *definite* ratio to the prism, its content is easily ascertained; and the same applies to all solids, the ratio of which to their enveloping cylinder or prism is known. But when we come to the frustum the case is altered. Let a plane, parallel to the base, and half-way between it and the vertex, pass through the prism and its inscribed pyramid; half the prism is now cut off, but not so half the pyramid. Solids being as the cubes of their like dimensions, only one-eighth of the pyramid is removed, and seven-eighths remain. Divide the topmost part of each solid thus cut off into segments, by planes passing through each, parallel to the base, and with these commence to build up the prism and the pyramid anew. With every increment we find the ratio between the frustum and its investing prism disturbed. But, of course, at each pause in the

process, there is a mean section somewhere in the frustum, such, that a prism passing through it, of equal height with the frustum, would be equal to that frustum. But where is this mean section to be found? And what is the ratio of its side to the side of the base? It is evidently not midway between the base and the top of the frustum—that is, not the arithmetical mean, as we have seen in Question 199; in which, if the side of the mean section were 12, (the arithmetical mean between the top and the bottom,) the solid would be a cube; but we find it to be greater. Nor is it the geometrical mean, for this would be still less, and raise it higher. The mean sectional area, therefore, in tapering frustums, is a little below the intermediate section; and its diameter, of course, in excess of the intermediate diameter.

503. A formula for the content of the frustum is easily derived by finding, first, the content of the entire pyramid; and, secondly, that of the segmental pyramid; their difference is, of course, the content of the frustum. Putting h the height of the frustum, S the side of its base, and s of its top, or the base of the segmental pyramid, we have

$\mathrm{S} - s : s :: h : \dfrac{h\,s}{\mathrm{S} - s}$, the height of the segmental pyramid;

$$\therefore \frac{h\,s^3}{\mathrm{S} - s} \cdot \frac{1}{3\,k} = \text{its content; and}$$

$\left(h + \frac{h\,s}{S - s}\right) S^2 \cdot \frac{1}{3\,k}$ = the content of the entire pyramid.

Deducting the former from the latter, gives

$$\frac{h(S^3 - s^3)}{3\,k(S - s)} = \frac{h(S^2 + S\,s + s^2)}{3\,k}$$

the content of the frustum.

504. As the pyramid is to its enveloping prism as 1 : 3, the frustum is, necessarily, in a ratio between these. It may run up till it loses itself in the complete solid, and becomes as 1 : 3; it may sink down till it falls into the plane, and is as 3 : 3. Consequently, it shifts up and down with every variety of ratio between the limits of 1 and 3.

The formula for the frustum is

$$C = \frac{h(S^2 + S\,s + s^2)}{3\,k}$$

Bring the S^2 outside the parenthesis, and we have

$$C = \frac{h\,S^2\left(1 + 1\,\frac{s}{S} + 1\,\frac{s^2}{S^2}\right)}{(1 + 1 + 1)k}$$

Putting, therefore, Σ as the side of the mean sectional area; and, consequently, Σ^2 as such area, then

$1 + 1\,\frac{s}{S} + 1\,\frac{s^2}{S^2} : 1 + 1 + 1 :: \Sigma^2 : S^2 \therefore$ the mean sectional area : area of the base :: the frustum : its enveloping prism.

EXAMPLE.

505. Suppose the frustum of a square prism to be 20 inches deep, the side of the greater end 22, of the less 16: required the side of a square prism that, of equal height with the frustum, shall be equal to the frustum; also its content in gallons.

Here $\frac{s}{S} = \frac{8}{11}$ $\therefore$ $\frac{s^2}{S^2} = \frac{64}{121}$; also $22^2 = 484$

Then $1 + \frac{8}{11} + \frac{64}{121} : 1 + 1 + 1 :: 2\frac{31}{121} : 3 ::$ $273 : 363$; and $363 : 273 :: 484 : 364$, the mean sectional area;

$\therefore \sqrt{364} = 19.078784$, side of equal square prism; and $364 \times 20 = 7280$, solidity of frustum;

$\frac{16 + 22}{2} = 19$, side of intermediate square; and $19^2 = 361$, intermediate section;

$364 - 361 = 3$, the difference between the mean section and the intermediate section;

$22 - 16 = 6$, the difference between the top and bottom sides;

$6^2 = 36$, square of the same;

And 3, above found, $= \frac{1}{12}$ of 36; that is, the difference between the mean section and the intermediate section is $\frac{1}{12}$ of the square of the difference of the top and bottom sides; which may be shown as follows:—Put h the less side, and d the difference between it and the greater; then $h + d =$ the greater; and $h + \frac{d}{2} =$ the intermediate side. Now,

$$\begin{array}{lllr}
\text{Top} & \overline{h}|^2 & = & h^2 \\
\text{Bottom} & h + d & = & h^2 + 2\,h\,d + d^2 \\
\text{Sum} & 2\,h + d & = & 4\,h^2 + 4\,h\,d + d^2 \\
 & & & 6|\overline{6\,h^2 + 6\,h\,d + 2\,d^2} \\
 & & & h^2 + h\,d + \tfrac{1}{3}\,d^2 \text{ mean area.} \\
 & \overline{h + \frac{d}{2}}|^2 & = & h^2 + h\,d + \tfrac{1}{4}\,d^2 \text{ intermediate area.} \\
 & & & \tfrac{1}{12}\,d^2 \text{ difference.}
\end{array}$$

Numerically—

$$\begin{array}{lllr}
\text{Top} & \overline{16}|^2 & = & 256 \\
\text{Bottom} & 22 & = & 484 \\
\text{Sum} & 38 & = & 1444 \\
 & & & 6|\overline{2184} \\
 & & & 364 \text{ mean area.} \\
 & 19^2 & = & 361 \text{ intermediate area.} \\
 & & & 3 \text{ difference.}
\end{array}$$

$364 \times 20 = 7280$, solidity of frustum;

$484 \times 20 = 9680$, solidity of enveloping prism;

And $2\frac{31}{121} : 3 :: 273 : 363 :: 364 : 484 ::$ mean sectional area : area of base :: 7280 : 9680 :: solidity of frustum : solidity of circumscribing prism.

Also $\frac{7280}{277.274} = 26.25$, content of frustum in gallons.

506. Hence, when the side of the less base is $\frac{8}{11}$ of the greater, the content of the frustum is $\frac{273}{363}$ of its enveloping prism. ∴ As the content of the prism is $\frac{h\,s^2}{k}$, the content of the frustum will be $\frac{273\,h\,s^2}{363\,k}$.

$$= h\,s^2\,\frac{273}{363 \times 277.274}$$

$$= h\,s^2\,\frac{273}{100650.462}$$

$$= h\,s^2\,\frac{1}{368.682}$$

$$= h\,s^2\,\frac{1}{19.2^2}$$

$$= h\,s^2\,.0027123$$

Hence the formulæ, for the content of the frustum in gallons, are

$$\frac{h\,s^2}{368.682}\ ;\ h\,s^2\,.0027123\ ;\ \frac{h\,s^2}{19.2^2}$$

$$20 \times 22^2 \left\{ \begin{array}{l} \div\ 368.682 \\ \times\ .0027123 \end{array} \right\} = 26.25 \text{ gallons.}$$

By the slide-rule—

$$\frac{20 \times 22}{19.2^2} = ?$$

19.2 D : 20 C :: 22 D : 26.2 C

507. We are thus enabled to place the angular frustum within the square prism, and the round frustum within the cylinder, as we did the complete solids; to determine the ratio, in any particular case, existing between them; and, consequently, to exhibit the rule, however varied, for expressing the content of every solid, complete or frustum, under the one uniform appearance of $\frac{\mathrm{L\,S}^2}{\mathrm{V}\,k}$ or $\frac{\mathrm{L\,D}^2}{\mathrm{V}\,c}$; where k represents the number of cubic, c the number of cylindrical inches in a gallon, and V a multiple, varying with the ratio of S to s and D to d.

508. Dr. Hutton's formula is

$$\frac{\mathrm{L}(39\,\mathrm{B}^2 + 26\,\mathrm{B\,H} + 25\,\mathrm{H}^2)}{(39 + 26 + 25)c}$$

Bring the B² outside the parenthesis, and it becomes

$$\mathrm{C} = \frac{\mathrm{L\,B}^2\left(39 + 26\,\frac{\mathrm{H}}{\mathrm{B}} + 25\,\frac{\mathrm{H}^2}{\mathrm{B}^2}\right)}{(39 + 26 + 25)c}$$

509. Let it now be required to transform this into the expression $\frac{\mathrm{L\,B}^2}{\mathrm{V}\,c}$; we have simply to cancel both numerator and denominator by the parenthetical coefficient, or submultiple, of the former—that is,

$$\mathrm{V} = 90 \div \left\{39 + 26\,\frac{\mathrm{H}}{\mathrm{B}} + 25\,\frac{\mathrm{H}^2}{\mathrm{B}^2}\right\}$$

As $c = 353.036$;

$$C = L\,B^2\left(39 + 26\,\frac{H}{B} + 25\,\frac{H^2}{B^2}\right)\frac{1}{90 \times 353.036}$$

510. To apply this to a particular case, let us take the cask before mentioned, in which $L = 40, B = 32, H = 24$, and $\therefore \frac{H}{B} = \frac{3}{4}$; we then have

$$C = L\,B^2\left(39 + \overline{26 \times \tfrac{3}{4}} + \overline{25 \times \tfrac{9}{16}}\right)\frac{1}{31773.24}$$

$$= L\,B^2\left(\frac{624 + 312 + 225}{16}\right)\frac{1}{31773.24}$$

$$= L\,B^2\,\frac{1161}{508371.84}$$

$$= L\,B^2\,\frac{1}{437.874}$$

$$= L\,B^2\,\frac{1}{20.92^2}$$

$$= L\,B^2\,.002283759$$

511. These, then, are the divisor, gauge point, and factor, for a cask of Hutton's variety, when the head is $\frac{3}{4}$ of the bung.

Formulæ for Gallons.

$$\frac{L\,B^2}{437.874}\,;\ L\,B^2\,.002283759\,;\ \frac{L\,B^2}{20.92^2}$$

$$40 \times 32^2\left\{\begin{matrix} \div\ 437.874 \\ \\ \times\ .002283759 \end{matrix}\right\} = 93.54 \text{ gallons.}$$

By the slide-rule—

$$\frac{40 \times 32^2}{20.92^2} = ?$$

$$20.92 \text{ D} : 40 \text{ C} :: 32 \text{ D} : 93.5 \text{ C}$$

512. An extreme case of a protuberant cask is when the bung is double the head, and, consequently, the quotient of the latter by the former = ½ or .5. In this case the formula becomes

$$\text{C} = \text{L B}^2 \left(39 + \overline{26 \times \tfrac{1}{2}} + \overline{25 \times \tfrac{1}{4}}\right) \frac{1}{31773.24}$$

$$= \text{L B}^2 \left(\frac{156 + 52 + 25}{4}\right) \frac{1}{31773.24}$$

$$= \text{L B}^2 \frac{233}{127092.96}$$

$$= \text{L B}^2 \frac{1}{545.4633}$$

$$= \text{L B}^2 \frac{1}{23.35^2}$$

$$= \text{L B}^2 \, .0018333$$

513. These, then, are the divisor, gauge point, and factor, for a cask of Hutton's variety, when the head divided by the bung = .5.

514. An extreme case of a cask the other way is when it becomes a cylinder, and the head divided by the bung = 1.

In this case the formula becomes

$$C = \frac{L\,B^2\,(39 + 26 + 25)}{(39 + 26 + 25)\;353.036}$$

$$= \frac{L\,B^2}{353.036},$$ the well-known rule for the cylinder.

515. We may thus, between the extreme case, in which the bung is double the head, and the quotient .5; and the cylinder, in which the bung is equal to the head, and the quotient 1; compute a table of divisors, factors, and gauge points, for every intermediate variety, commencing with .50, and proceeding to .51; .52; .53; &c.; up to unity.

516. Such table being computed, to find the content of any cask of Dr. Hutton's variety, we have simply to divide the head by bung, the look in the table for the quotient, and against it we shall find the divisor, factor, and gauge point, with which we may proceed to compute the content, from the bung diameter alone.

517. Instead of this, however, we may mark the numbers on the Slide-rule, opposite to the quotients, and so save ourselves the trouble of looking into the table.

518. When the cask is a cylinder—that is, when $\frac{H}{B} = 1$. the gauge point is 18.78. When it is so protuberant that the bung is double the head—that is,

when $\frac{H}{B} = .5$, the gauge point has been shown to be 23.35.

519. Let the end of the A line be accounted unity, or 1. then the numbers upon it, running back, will be .9; .8; .7; &c. This being premised, over it lay down a line of gauge points, running backwards, commencing with 18.78 over the 1 of A, and ending with 23.35 over .5 of A, and fill up the intermediate numbers from the table.

To apply this line, suppose we refer to the cask before mentioned, where B = 32, and H = 24.

Under 24 of A set 32 of B; then over 1 of B is .75, the *quotient* on A; and over *that* stands 20.92 on the variety line, the equicylindric gauge point for computing the content of the cask from the bung diameter.

520. This line is marked on the rule X.... H' V, Extra line, Hutton's variety.

521. Against it is a line marked X .... Y' V, Young's variety, graduated in a similar manner from a hypothetical cask of Dr. Young's, which gives the content somewhat greater than Dr. Hutton's.

522. From the foregoing observations, it is evident that a table may be prepared for *every* formula; and, consequently, that lines may be laid down for the four varieties of casks, the frustums of the prolate spheroid, parabolic spindle, paraboloid, and cone.

523. The first variety is the middle frustum of the prolate spheroid, the rule for which is

$$C = \frac{L(2\ B^2 + H^2)}{1059.108}$$

Taking the same cask as before, we have

$$C = \frac{L\ B^2\left(2 + 1\,\frac{H^2}{B^2}\right)}{(2 + 1)\ 353.036}$$

$$= L\ B^2\left(2 + \frac{9}{16}\right)\frac{1}{1059.108}$$

$$= L\ B^2\left(\frac{32 + 9}{16}\right)\frac{1}{1059.108}$$

$$= L\ B^2\ \frac{41}{16945.728}$$

$$= L\ B^2\ \frac{1}{413.31}$$

$$= L\ B^2\ \frac{1}{20.33^2}$$

$$= L\ B^2\ .0024195$$

Hence the formulæ for computing the content of a cask of the 1st variety, when $\frac{H}{B} = .75$, are

$$\frac{L\ B^2}{413.31}\ ;\ L\ B^2\ .0024195\ ;\ \frac{L\ B^2}{20.33^2}$$

$$40 \times 32^2 \left\{\begin{array}{l} \div\ 413.31 \\ \times\ .0024195 \end{array}\right\} = 99.1 \text{ gallons.}$$

By the slide-rule—

$$\frac{40 \times 32^2}{20.33^2} = ?$$

20.33 D : 40 C :: 32 D : 99.1 C

524. Compute a table, as before, for every intermediate value of $\frac{H}{B}$ from .5 to 1.; place the numbers, denoting the gauge points, on the rule, opposite the respective quotients, and mark them X .... 1' V, a line for the 1st variety, or frustum of prolate spheroid.

525. The next variety is the middle frustum of the parabolic spindle, the rule for which, as has been shown, may be put under the form

$$C = L(8\,B^2 + 4\,B\,H + 3\,H^2)\frac{1}{5295.54}$$

526. Taking the former cask, in which $\frac{H}{B} = \frac{3}{4}$, and applying it to the formula,

$$C = \frac{L\,B^2\left(8 + 4\frac{H}{B} + 3\frac{H^2}{B^2}\right)}{(8 + 4 + 3)\;353.036}$$

we have

$$C = L\,B^2\left(8 + 4\frac{H}{B} + 3\frac{H^2}{B^2}\right)\frac{1}{5295.54}$$

$$= L\,B^2\left(8 + \overline{4 \times \frac{3}{4}} + \overline{3 \times \frac{9}{16}}\right)\frac{1}{5295.54}$$

$$= L\,B^2\left(8 + 3 + \frac{27}{16}\right)\frac{1}{5295.54}$$

$$= L\,B^2 \left(\frac{128 + 48 + 27}{16}\right) \frac{1}{5295.54}$$

$$= L\,B^2 \frac{203}{84728.64}$$

$$= L\,B^2 \frac{1}{417.382}$$

$$= L\,B^2 \frac{1}{20.43^2}$$

$$= L\,B^2\ .0023959$$

527. Hence the formulæ for computing the content of a cask of the 2nd variety, when $\frac{H}{B} = .75$, are

$$\frac{L\,B^2}{417.382}\ ;\ L\,B^2\ .0023959\ ;\ \frac{L\,B^2}{20.43^2}$$

$$40 \times 32^2 \left\{ \begin{matrix} \div\ 417.382 \\ \times\ .0023959 \end{matrix} \right\} = 98.13 \text{ gallons.}$$

By the slide-rule—

$$\frac{40 \times 32^2}{20.43^2} = ?$$

$$20.43\ D : 40\ C :: 32\ D : 98.13\ C$$

528. Compute a table, as before, for every intermediate value of $\frac{H}{B}$ from .5 to 1.; place the numbers, denoting the gauge points, on the rule, opposite the respective quotients, and mark them X .... 2^d V, a

line for the 2nd variety, or middle frustum of parabolic spindle.

529. The third variety is composed of the frustums of two paraboloids, the rule for which is

$$C = \frac{L(B^2 + H^2)}{706.072}$$

Applied with the same cask, it becomes

$$C = \frac{L\,B^2\left(1 + 1\,\frac{H^2}{B^2}\right)}{(1 + 1)\;353.036}$$

$$= L\,B^2\left(1 + \frac{9}{16}\right)\frac{1}{706.072}$$

$$= L\,B^2\left(\frac{16 + 9}{16}\right)\frac{1}{706.072}$$

$$= L\,B^2\,\frac{25}{11297.152}$$

$$= L\,B^2\,\frac{1}{451.886}$$

$$= L\,B^2\,\frac{1}{21.25^2}$$

$$= L\,B^2\,.00221294$$

530. Hence the formulæ for computing the content of a cask of the 3rd variety, when $\frac{H}{B} = .75$, are

$$\frac{L\,B^2}{451.886}\;;\;L\,B^2\,.00221294\;;\;\frac{L\,B^2}{21.25^2}$$

$$40 \times 32^2 \left\{ \begin{array}{l} \div\ 451.886 \\ \times\ .00221294 \end{array} \right\} = 90.64 \text{ gallons.}$$

By the slide-rule—

$$\frac{40 \times 32^2}{21.25^2} = ?$$

$$21.25\ \text{D} : 40\ \text{C} :: 32\ \text{D} : 90.64\ \text{C}$$

531. Compute a table, as before, for every intermediate value of $\frac{H}{B}$ from .5 to 1.; place the numbers, denoting the gauge points, on the rule, opposite the respective quotients, and mark them X .... 3^d V, a line for the 3rd variety, or frustums of two paraboloids.

532. The fourth and last variety is composed of the frustums of two cones, the rule for which, in its simplest form, is

$$C = \frac{L(B^2 + B\,H + H^2)}{1059.108}$$

Applied to the same cask, it becomes

$$C = \frac{L\,B^2\left(1 + 1\frac{H}{B} + 1\frac{H^2}{B^2}\right)}{(1 + 1 + 1)\,353.036}$$

$$= L\,B^2\left(1 + \frac{3}{4} + \frac{9}{16}\right)\frac{1}{1059.108}$$

$$= L\,B^2\left(\frac{16 + 12 + 9}{16}\right)\frac{1}{1059.108}$$

$$= \text{L B}^2 \frac{37}{16945.728}$$

$$= \text{L B}^2 \frac{1}{457.992}$$

$$= \text{L B}^2 \frac{1}{21.4^2}$$

$$= \text{L B}^2 \, .0021834$$

533. Hence the formulæ for computing the content of a cask of the 4th variety, when $\frac{H}{B} = .75$, are

$$\frac{\text{L B}^2}{457.992} \; ; \; \text{L B}^2 \, .0021834 \; ; \; \frac{\text{L B}^2}{21.4^2}$$

$$40 \times 32^2 \left\{ \begin{matrix} \div \; 457.992 \\ \times \; .0021834 \end{matrix} \right\} = 89.43 \text{ gallons.}$$

By the slide-rule—

$$\frac{40 \times 32^2}{21.42^2} = ?$$

21.42 D : 40 C :: 32 D : 89.4 C

534. Compute a table, as before, for every intermediate value of $\frac{H}{B}$ from .5 to 1.; place the numbers denoting the gauge points, on the rule, opposite to the respective quotients, and mark them X . . . . 4^{h} V, a line for the 4th variety, or frustums of two cones.

535. Of course, there was no necessity for selecting the cask with the bung 32 and the head 24; the ratio between them, viz., 1 : $\frac{3}{4}$, was all that was required; but it was conceived that an illustration by a real cask would render the explanation more intelligible.

536. The idea of these variety lines is exceedingly ingenious. Mr. Lowry, it is said, was the first to show the mode of calculating the divisors, factors, and gauge points, for Dr. Hutton's variety; and Mr. Woolgar the first to propose them for the four others, and to lay them down on the Slide-rule.

537. It is not usual to construct X lines for the 3rd and 4th varieties, as they are of no practical utility. The following calculations are for every fifth number of the 1st and 2nd varieties:—

1st variety.

$\overline{2+H\div B}\|^2$				
2+.5	=2.25	1059.108÷2.25	=21.69	over .5
.55	=2.3025	2.3025	=21.44	.55
.6	=2.36	2.36	=21.18	.6
.65	=2.4225	2.4225	=20.90	.65
.7	=2.49	2.49	=20.62	.7
.75	=2.5625	2.5625	=20.33	.75
.8	=2.64	2.64	=20.03	.8
.85	=2.7225	2.7225	=19.72	.85
.9	=2.81	2.81	=19.41	.9
.95	=2.9025	2.9025	=19.10	.95

2nd variety.

$8 + 4.\overline{H \div B} + 3.\overline{H \div B}|^2$

$8 + \overline{4 \times .5} + \overline{3 \times .25} = 10.75$	5295.54 ÷	$10.75 = 22.19$	over .5
$4 \times .55 + 3 \times .3025 = 11.1075$		$11.1075 = 21.83$	.55
$4 \times .6 + 3 \times .36 = 11.48$		$11.48 = 21.47$	.6
$4 \times .65 + 3 \times .4225 = 11.8675$		$11.8675 = 21.12$	.65
$4 \times .7 + 3 \times .49 = 12.27$		$12.27 = 20.77$	.7
$4 \times .75 + 3 \times .5625 = 12.6875$		$12.6875 = 20.43$	.75
$4 \times .8 + 3 \times .64 = 13.12$		$13.12 = 20.09$	.8
$4 \times .85 + 3 \times .7225 = 13.5675$		$13.5675 = 19.75$	.85
$4 \times .9 + 3 \times .81 = 14.03$		$14.03 = 19.42$	.9
$4 \times .95 + 3 \times .9025 = 14.5075$		$14.5075 = 19.10$	.95

538. As the differences between these vary but little, the rest may safely be interpolated. For instance, selecting a portion of the 2nd variety,

Over .6	is	21.47	
.65		21.12	35 difference.
.7		20.77	35
.75		20.43	34
.8		20.09	34

539. Now, as the second difference is the same as the first, and the third is only 1 less than the second, we take $\frac{1}{5}$ of 35, viz., 7, for the four intermediate differences between 21.47 and 21.12; and 21.12 and 20.77; and they accordingly stand thus—

Over .6	is	21.47
.61		21.40
.62		21.33
.63		21.26
.64		21.19
.65		21.12
.66		21.05
.67		20.98

540. Before, however, we can construct a line by laying down one *against* another, a farther preparation is necessary. Had we to place the 6 under 21.47, it could easily be done; but we cannot so readily place 21.47 over 6. The numbers on the right hand must not contain more than one place of decimals; for this purpose, therefore, proportional differences must be taken, as with tables of logarithms. 21.33 falls over .62, and 21.26 over .63. Over what does 21.3 fall?

$$\begin{array}{rcr} 21.33 &=& .62 \\ 21.26 &=& .63 \\ \hline 7 &=& -.01 \end{array} \qquad\qquad \begin{array}{r} 21.3 \\ 21.26 \\ \hline 4 \end{array} = ?$$

$$-.01 \times \frac{4}{7} = -.005$$

$$\text{And} \quad .63 - .005 = .625$$

Hence 21.3 is to be placed over .625.

Again, over what does 21.2 fall?

$$\begin{array}{rcr} 21.26 &=& .63 \\ 21.19 &=& .64 \\ \hline 7 &=& -.01 \end{array} \qquad\qquad \begin{array}{r} 21.2 \\ 21.19 \\ \hline 1 \end{array} = ?$$

$$-.01 \times \frac{1}{7} = -.001$$

$$\text{And} \quad .64 - .001 = .639$$

Hence 21.2 falls over .639

And, thus proceeding, we find

21.5	over	.597,	whose logarithm is	775
21.4		.610,		785
21.3		.625,		795
21.2		.639,		805
21.1		.653,		815
21.		.667,		824

541. Then, supposing we have the A line already laid down, and wish to construct, immediately over it, this 2nd variety line, we carefully look out 597 on the A line; and, with the help of an **L** square and a finely-pointed pencil, draw over it a mark for the number 21.5; over 610 another for 21.4; and so on.

542. This, when we have a logarithmic line already laid down, is far the readiest mode of constructing a variety line; but if it be desired to lay down the variety line independently of the A line, then from a scale of equal parts, containing 1000 to the intended radius, we select 775, the corresponding logarithm, and proceed in the manner described at page 5. For taking the logarithms, however, more correctly, it would be advisable to extract the square roots to four places of decimals.

543. As these lines are all employed in the same way, it will be sufficient to show the mode of using them for the 1st variety.

EXAMPLES.

544. The length of a mountain butt is 41 inches, the head 26.6, bung 32: required the content.

Under 26.6 A set 32 B; then over 1 is 19.8, the equicylindric gauge point for 1st variety.

$$\text{Then } \frac{41 \times 32^2}{19.8^2} = ?$$

19.8 D : 41 C :: 32 D : 107 C

545. The length of a Lisbon hogshead is 29 inches, the head 25, bung 28: required the content.

Under 25 A set 28 B; over 1 is 19.46, equicylindric gauge point for 1st variety.

$$\text{Then } \frac{29 \times 28^2}{19.46^2} = ?$$

19.46 D : 29 C :: 28 D : 60 C, 1st variety.

546. The length of a Spanish red hogshead is 30 inches, head 21, bung 24, content?

Under 21 A set 24 B; over 1 is 19.56, equicylindric gauge point for 1st variety.

$$\text{Then } \frac{30 \times 24^2}{19.56^2} = ?$$

19.56 D : 30 C :: 24 D : 45 C, 1st variety.

547. A Charente brandy puncheon is 46 inches long, 26.5 head diameter, 31 bung, content?

Under 26.5 A set 31 B; over 1 is 19.68, equicylindric gauge point for 1st variety.

$$\text{Then } \frac{46 \times 31^2}{19.68^2} = ?$$

19.68 D : 46 C :: 31 D : 114 C, 1st variety.

548. As casks appear, in general, to be composed of portions of two varieties, is it not possible to combine portions of two of the formulæ, in a manner that shall be simpler than Dr. Hutton's, and, at the same time, suited for the purposes of the Slide-rule, which, in all cases of practical cask gauging, appears to be the great desideratum?

549. The divisor for the 1st variety is the same as for the 2nd. Omitting the latter, then, let the others be grouped together as before; they are

$$\frac{L(2.B^2 + H^2)}{1059.108};\ \frac{L(B^2 + H^2)}{706.072};\ \frac{L(B^2 + H^2 + \overline{B + H}^2)}{2118.216}$$

550. Multiply the numerator and denominator of the first by 2, and of the second by 3, and we have

$$\frac{2\,L(2.B^2 + H^2)}{2118.216};\ \frac{3\,L(B^2 + H^2)}{2118.216};\ \frac{L(B^2 + H^2 + \overline{B + H}^2)}{2118.216}$$

For the Slide-rule these become

1st. Pro. sphd. $\dfrac{2\,L(2.B^2 + H^2)}{46^2}$

2nd. Par. spin. $\dfrac{2\,L(2.B^2 + H^2 - \frac{1}{10}\text{ of }.\overline{2\text{ diff.}}^2)}{46^2}$

3rd. Two parab. $\dfrac{3\,L(B^2 + H^2)}{46^2}$

4th. Two co. $\dfrac{L(B^2 + H^2 + \overline{B + H}^2)}{46^2}$

Or, dividing numerator and denominator by 4,

1st. Pro. sphd. $\dfrac{\frac{1}{2}\,L(2.B^2 + H^2)}{23^2}$

2nd. Par. spin. $\dfrac{\frac{1}{2}\,L(2.B^2 + H^2 - \frac{1}{10}\text{ of }.\overline{2\text{ diff.}}^2)}{23^2}$

3rd. Two parab. $\dfrac{\frac{3}{4}\,L(B^2 + H^2)}{23^2}$

4th. Two co. $\dfrac{\frac{1}{4}\,L(B^2 + H^2 + \overline{B + H}^2)}{23^2}$

551. All gauge points are readily shown on the Slide-rule. Thus, the ratios given for the cylinder, spheroid, parabolic spindle, paraboloid, cone, and round frustum, are 1, $\frac{2}{3}$, $\frac{8}{15}$, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{6}$. Inverting them, we have 1, $\frac{3}{2}$, $\frac{15}{8}$, $\frac{2}{1}$, $\frac{3}{1}$, $\frac{6}{1}$. Multiplying by 8, to clear them of fractions, they become 8, 12, 15, 16, 24, 48. The gauge point for the cylinder is 18.78. Bring the 8 of C over this, then under the other numbers stand their gauge points; thus,

C	8	12	15	16	24	48
D	18.78	23	25.72	26.57	32.54	46

552. Hence the cask formerly given may be exhibited as follows:—

1st. Pro. sphd.

$$\frac{\begin{array}{c|c|c}\frac{1}{2}L & L & 2L\\ 20 & 40 & 80\end{array}}{\begin{array}{c|c|c}23^2 & 32.54^2 & 46^2\end{array}}\left\{\begin{array}{c|l}\overline{\quad}^2 & \\ 32 & =38.7\\ \cdot & =38.7\\ 24 & =21.7\\ & \overline{99.1}\text{ gallons.}\end{array}\right.$$

2nd. Par. spin.

$$\frac{\begin{array}{c|c|c}\frac{1}{2}L & L & 2L\\ 20 & 40 & 80\end{array}}{\begin{array}{c|c|c}23^2 & 32.54^2 & 46^2\end{array}}\left\{\begin{array}{c|l}\overline{\quad}^2 & \\ 32 & =38.7\\ \cdot & =38.7\\ 24 & =21.7\\ \overline{\;8\;} & \overline{99.1}\\ \frac{1}{10}\ \overline{16} & =\quad .97\\ & \overline{98.13}\text{ gallons.}\end{array}\right.$$

3rd. Two parab.

$$\frac{\begin{array}{c|c|c}\frac{3}{4}L & L & 3L\\ 30 & 40 & 120\end{array}}{\begin{array}{c|c|c}23^2 & 26.57^2 & 46^2\end{array}}\left\{\begin{array}{c|l}\overline{\quad}^2 & \\ 32 & =58.1\\ 24 & =32.6\\ & \overline{90.7}\text{ gallons.}\end{array}\right.$$

4th. Two co.

$$\frac{\begin{array}{c|c|c}\frac{1}{4}L & \frac{1}{2}L & L\\ 10 & 20 & 40\end{array}}{\begin{array}{c|c|c}23^2 & 32.54^2 & 46^2\end{array}}\left\{\begin{array}{c|l}\overline{\quad}^2 & \\ 32 & =19.4\\ 24 & =10.87\\ 56 & =59.2\\ & \overline{89.47}\text{ gallons.}\end{array}\right.$$

553. As the formula for the prolate spheroid gives the content too great, and the one for the paraboloid gives it too little, the object desired to be attained seems to be, not so much to consider the cask as made up of precisely distinct lengths of two separate solids, as to devise a rule that shall give, in some definite ratio, a proportional between the two contents.

554. Of course, every contrivance of this kind labours under the imputation of being purely arbitrary and hypothetical; but the same objection applies equally to considering the cask as the frustum of the prolate spheroid, which, it is allowed on all hands, gives the content too great, and requires a no less arbitrary deduction from the length to compensate for the error of protuberance towards the extremities.

555. The formula which, after much careful consideration, and verification with tables of casks that have actually been gauged and measured, appears to me the best calculated to give the nearest approximation to real casks, is deduced from conceiving them made up of parts of the prolate spheroid and paraboloid; not portions of the length, as in Dr. Hutton's method, but portions of the capacities of the two solids, as follows:—

$$\tfrac{1}{5}\,\begin{matrix}\textit{frustum of}\\ \textit{paraboloid}\end{matrix} + \tfrac{3}{5}\,\begin{matrix}\textit{frustum of}\\ \textit{prolate spheroid}\end{matrix} + \tfrac{1}{5}\,\begin{matrix}\textit{frustum of}\\ \textit{paraboloid}\end{matrix}$$

556. The content of the frustum of the prolate

spheroid is $\frac{2\,L(2\,B^2 + H^2)}{2118.216}$; and of the paraboloid $\frac{3\,L(B^2 + H^2)}{2118.216}$.

Disregarding the denominator and the L of the numerator, as common to both, the

$$\begin{aligned} &\text{Spheroid is} && 2\,H^2 + 4\,B^2 \\ &\text{Paraboloid} && 3\,H^2 + 3\,B^2 \end{aligned}$$

Consequently,

$$\begin{aligned} \tfrac{3}{5}\text{ spheroid} &= 1\tfrac{1}{5}\,H^2 + 2\tfrac{2}{5}\,B^2 \\ \tfrac{2}{5}\text{ paraboloid} &= \underline{1\tfrac{1}{5}\,H^2 + 1\tfrac{1}{5}\,B^2} \\ \text{Sum} &= \underline{2\tfrac{2}{5}\,H^2 + 3\tfrac{3}{5}\,B^2} \end{aligned}$$

Restore the length and the denominator, and we have

$$\begin{aligned} \text{Content} &= \frac{L(2\tfrac{2}{5}\,H^2 + 3\tfrac{3}{5}\,B^2)}{2118.216} \\ &= \frac{L(2\tfrac{2}{5}\,H^2 + 3\tfrac{3}{5}\,B^2)}{(2\tfrac{2}{5} + 3\tfrac{3}{5})353.036} \\ &= \frac{L(1\,H^2 + 1\tfrac{1}{2}\,B^2)}{(1 + 1\tfrac{1}{2})353.036} \\ &= \frac{L(H^2 + 1\tfrac{1}{2}\,B^2)}{882.59} \\ &= \frac{L(H^2 + 1\tfrac{1}{2}\,B^2)}{29.7^2} \\ &= L(H^2 + 1\tfrac{1}{2}\,B^2)\ .001133 \end{aligned}$$

557. For facility of computation, both with the factor and gauge point, which latter occurs on a part of the D line where it can be distinctly taken and carefully marked, this formula is surpassed by none; and I cannot help thinking it will be found as nearly correct as it is possible to devise a general rule for so indefinite a figure as a cask.

EXAMPLES.

558. A cask is 40 inches long, head diameter 24, and bung 32: required its content.

$$\begin{array}{rcrcr}
H^2 & = & 24^2 & = & 576 \\
B^2 & = & 32^2 & = & 1024 \\
 & & \frac{1}{2} & = & 512 \\
\hline
 & & & & 2112 \times 40 \left\{ \begin{array}{l} \div\ 882.59 \\ \\ \times\ .001133 \end{array} \right\} = 95.7 \text{ gallons.}
\end{array}$$

By the slide-rule—

Parabolic spheroidal cask $\quad \dfrac{40}{29.7^2} \left\{ \begin{array}{c} 24 \\ 32 \end{array} \right]^2 \begin{array}{l} = 26.1 \\ = 46.4 \end{array}$

$$\begin{array}{rr}
\frac{1}{2} = & 23.2 \\
\hline
 & 95.7 \\
\hline
\end{array}$$

559. The same cask, otherwise computed, is as follows:—

	Varieties.	Content.
1st.	Prolate spheroid	99.1
2nd.	Parabolic spindle	98.13
	Young's	96.18
	Parabolic spheroidal	95.7
	Hutton's	93.54
3rd.	Two paraboloids	90.64
4th.	Two cones	89.41

560. A cask is 44 inches long, the bung diameter 42, and the head 38 inches: required the content by the proposed rule.

Parabolic spheroidal $$\frac{44}{29.7^2}\left\{\left.\begin{array}{c}\overline{38}\\42\end{array}\right|^2\right.\begin{array}{l}=72\\=88\end{array}$$

$$\tfrac{1}{2} = 44$$

204 gallons.

561. A cask is 45 inches long, the bung diameter 36, and the head diameter 30 inches: required the content by the proposed rule.

Parabolic spheroidal $$\frac{45}{29.7^2}\left\{\left.\begin{array}{c}\overline{30}\\36\end{array}\right|^2\right.\begin{array}{l}=45.8\\=66.\end{array}$$

$$\tfrac{1}{2} = 33.$$

144.8 gallons.

562. A cask is 40 inches long, the head diameter is 30, and the bung 36 inches: required the content.

$$\text{Parabolic spheroidal}\quad \begin{array}{rl} \dfrac{40}{29.7^2}\left\{\begin{array}{c}\overline{30}\\36\end{array}\right|^2 & \begin{array}{l}= 40.8\\= 58.7\end{array}\\ \tfrac{1}{2} & = 29.3\\ \hline & 128.8 \text{ gallons.}\end{array}$$

563. A cask is 42 inches long, the head diameter is 28, the bung 35 inches: required the content.

$$\text{Parabolic spheroidal}\quad \begin{array}{rl} \dfrac{42}{29.7^2}\left\{\begin{array}{c}\overline{28}\\35\end{array}\right|^2 & \begin{array}{l}= 37.2\\= 58.3\end{array}\\ \tfrac{1}{2} & = 29.1\\ \hline & 124.6 \text{ gallons.}\end{array}$$

564. A cask is a cylinder 42 inches long, head and bung diameter 36 inches: required the content by this rule.

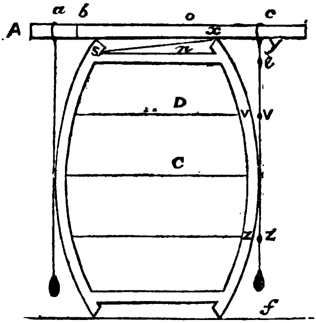
$$\begin{array}{rl} \dfrac{42}{29.7^2}\left\{\begin{array}{c}\overline{36}\\36\end{array}\right|^2 & \begin{array}{l}= 61.6\\= 61.6\end{array}\\ \tfrac{1}{2} & = 30.8\\ \hline & 154. \text{ gallons.}\end{array}$$

565. By the usual rule for the cylinder this is

$$\frac{42}{18.78^2}\Big\{36^2 = 154$$

$$18.78\,\text{D} : 42\,\text{C} :: 36\,\text{D} : 154\,\text{C}$$

566. If, in addition to the bung and head, we take the diameter half-way between the two, then the *true content* is readily found by the general rule for frustums. To take this intermediate diameter, proceed as follows:—Procure a piece of wood, A B, an inch square, and about 4 feet long; and near one end, as at *a*, cut a notch, and 2 inches from it, *b*, make a mark, and place the cipher 0. Then divide the distance from *b* to the end B into inches and tenths. Also procure two pieces of string, each with loops at one end, and heavy plummets of lead at the other. Before tying the loop, on one of the strings slip three pieces of cork, *e*, *v*, *z*; or, better still, three discs of india rubber, about $\frac{1}{8}$ of an inch thick, and $\frac{1}{2}$ an inch diameter. Then,



To take the dimensions of a standing cask—

567. Lay the rod A B over the middle of the head, and bring the plummet depending from *a* up to the

bulge of the cask. Then slip the other plummet along to *c*, till it touches the cask in like manner. The number now cut by *c* will be the internal bung diameter C, the distance *a b* of 2 inches, being an allowance for twice the thickness of the staves. With a measuring rod, take the distance from *y* (the *under* side of the rod A B) to the ground *f*. Also the distance from *o* (the *upper* side of the rod) to *n*, the head of the barrel. Then *y f*, minus twice *o n*, will be the internal length of the cask, the thickness of the square rod, A B, being supposed equal to the thickness of the head of the cask, which is generally 1 inch. To take the intermediate diameter, D, slip the top cork up to *e*, till the distance *y e* is equal to *o n*. The length of the cask being known, slip the second cork down to *v*, the distance *e v* being ¼ of the length; in the same manner adjust the cork *z*, if the cask is irregular. Then add together the distances *v v*, *z z*, and subtract their sum from the bung diameter; the remainder will be the intermediate diameter, D. But if the top and bottom of the cask be uniform, then only the cork at *v* need be used, and twice *v v* be deducted from the bung to get the intermediate diameter. The oblique line *s x*, measured from the inside of the chimb to the outermost sloped edge of the opposite stave, will be the internal head diameter; or twice the horizontal distance from *e* to the outside of the cask may be deducted from the bung.

568. Examples of casks by the intermediate diameter.

FORMULÆ.

$$\frac{L(H^2 + B^2 + \overline{2\,\delta}|^2)}{2118.216}$$

$$L(H^2 + B^2 + \overline{2\,\delta}|^2)\ .0004721$$

$$\frac{L(H^2 + B^2 + \overline{2\,\delta}|^2)}{46^2}$$

569. The length of a cask is 40 inches, the head 30, the bung 36, and the intermediate diameter 33.9 inches: required its content.

$$\begin{array}{l|r} \overline{30} & ^2 = 900 \\ 36 & = 1296 \\ 67.8 & = \underline{4596.84} \\ & \underline{6792.84} \end{array}$$

$$6792.84 \times 40 \left\{\begin{array}{l} \div\ 2118.216 \\ \times\ .0004721 \end{array}\right\} = 128.27 \text{ gallons.}$$

By the slide-rule—

$$\frac{40}{46^2}\left\{\begin{array}{l|r} \overline{30} & ^2 = 17. \\ 36 & = 24.5 \\ 67.8 & = \underline{86.8} \\ & \underline{128.3} \end{array}\right.$$

570. The length of a cask is 45 inches, the bung diameter 36, the head 30, and the intermediate 33.9: required its content.

$$\frac{45}{46^2}\left\{\begin{array}{l|} 30 \\ 36 \\ 67.8 \end{array}\right.^{2} \begin{array}{r} = 19.2 \\ = 27.6 \\ = 97.4 \\ \hline 144.2 \\ \hline \end{array}$$

571. In the practice of the Revenue, only three dimensions are taken, and all casks are considered as of the first variety, or frustums of prolate spheroids, the officers having private instructions to make an allowance or deduction in the length, according as the cask appears more or less protuberant.

572. For the purpose of taking the dimensions, they are furnished with four instruments, viz., a pair of long callipers, a pair of cross callipers, a bung rod, and a head rod.

573. *Head diameter.*—In using the head rod, the diameters of the two heads are to be taken, by placing the lower edge of the crooked brass, which is riveted to the end of the rod, within the chimb of the cask close to the head; then, bringing the projecting brass index, or finger piece, which will be found upon the slide, about $\frac{2}{3}$rds up the chimb, so as to be level with the groove where the head joins the staves,

and the mean thereof is to be considered as the head diameter.

574. *Bung diameter.*—Particular care is to be taken in ascertaining the bung diameter, it being of double the value of the other dimensions. First, with the cross callipers, which must be used upon all occasions, take the horizontal diameter in three different parts of the centre of the cask, and chalk down the mean, or average, as the cross, on the side of the bung, after deducting the thickness of the staves. Then, with the bung rod, examine the bottom and sides, to ascertain if there are any irregularities in the staves or form of the cask. If the cask appears fairly made, take the mean of the horizontal and perpendicular diameters for the bung diameter. Should two or three of the staves at the top or bottom of the cask appear flattened, and the rest of the cask circular, add to the perpendicular found as many tenths of an inch as the staves may be considered to be depressed, and then take the mean for the bung diameter. If, on the contrary, two or three of the staves at the bung appear forced up, so as to make what is termed a cocked bung, or the rod should have but little play at the bottom of the cask, deduct from the perpendicular found as many tenths of an inch as the case may seem to require, before taking the mean for the bung diameter. As deceptions are frequently made

use of to mislead the gauger, the greatest nicety of judgment is required in taking this dimension.

575. All casks of irregular thickness, such as rum puncheons, brandy puncheons, madeira pipes, and large-sized casks, it is necessary to bore occasionally, to ascertain the substance of the heads and staves. Bore the cask in the centre of the stave, at about one-third the distance from the head to the bung, and the head in the piece next the centre, and ascertain the thickness with the graduated brass stave gauge, or hook, prepared for the purpose. Cross and dip the casks as if they were an inch thick, and add or deduct from the bung thus found the mean difference of the actual thickness. For example, the staves of a rum puncheon, on boring, average $\frac{8}{10}$ of an inch thick, the two together make $1\frac{6}{10}$, but, as 2 inches have been allowed, this allowance is too much by $\frac{4}{10}$ of an inch; then, in dipping, to take the perpendicular diameter, 1 inch has been allowed, which allowance is $\frac{2}{10}$ too much; hence we have

Too little upon the cross 4 tenths.
" upon the perpendicular 2
2|6
.3 mean to be added.
Bung found 32.1
32.4 the bung diameter to be taken.

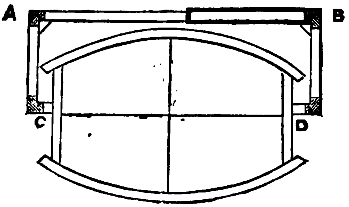
576. The staves upon a pipe of madeira average $\frac{12}{10}$; there is then

Too much upon the cross 4 tenths.
„ upon the perpendicular 2

$$\begin{array}{r} 2\,\overline{)6} \\ \hline .3 \end{array}$$
to be deducted.

28.7 bung found.
.3
28.4 bung diameter to be charged.

577. *Length.*—The length is to be taken with the long callipers; and to do this accurately the callipers are to be laid over the cask perfectly level. First, let the outer limb touch the back head, and then bring the inner one gently up to the front head, taking care not to press the rule. The length of the cask is shown on the stock, at once, with the long callipers, as they are so made as to throw off two inches for the supposed thickness of the two heads. This is not the case with the cross callipers. The cross callipers give the *external* diameter, the long callipers give the *internal* length.



The length is to be measured in two or three places, and the average to be taken.

578. To find the wet inches. Bring the lower end of the brass on the bung rod to the actual bung charged, if it be lowered; and, after wiping the rule, re-dip it into the cask, taking care to hold the brass firm against the rule, that it may not be forced up, and the true wet will be shown upon the rod. If the bung be raised, there must be added to the wet inches shown as many tenths as it is raised.

579. These Head and Bung Rods, Long and Cross Callipers, together with Sikes's Hydrometers, Saccharometers, Thermometers, Slide-rules, Glass Indexes or Tell Tales, Barometers, and every description of Optical and Mathematical Instrument, are sold by Messrs. Dring and Fage, Manufacturers by Appointment to Her Majesty's Honourable Board of Customs and Inland Revenue, established upwards of a century.

THE SPHEROID LINE.

580. The formula for the first variety is

$$\frac{L(2.B^2+\ H^2)}{32.54^2}$$

A cask is 32 inches long, the bung is 27, and the head 22.8: content?

$$\frac{32}{32.54^2}\left\{\begin{array}{c} \overline{22.8} \\ 27 \\ \cdot \end{array}\right|^2 \begin{array}{r} = 15.7 \\ = 22. \\ = 22. \\ \hline 59.7 \\ \hline \end{array}$$

581. Without the use of this formula, however, there is a contrivance for effecting the same object by a line marked SPH^{D}.

582. In laying down the variety line, formerly described, and marked as X .... 1' V, the cask was considered as standing within a cylinder, the diameter of which was equal to the diameter of the bung. The content was computed by the bung diameter, and the divisor was augmented accordingly, to make the necessary diminution. With the spheroid line the mean sectional area is supposed to be taken instead. A cylinder is imagined to pass up through this mean sectional area, and the content of the cask is computed with the usual formula for the cylinder, the divisor remaining unaltered, and the head being augmented to make the necessary increase. In other words, with the 1st variety line, we use the bung diameter, and increase the cylinder divisor; with the spheroid line, we retain the cylinder divisor, and decrease the bung diameter. The 1st variety line is mathematically correct; the spheroid line is not perfectly so, because the mean sectional area depends

upon the *ratio* of the head to the bung, and not upon the *difference* between the two; it is, however, sufficiently accurate for all practical purposes, and with so large a radius as is laid down on the head rod, it is very convenient.

583. If the head of a cask be 16 inches, and the bung 24, the difference is 8 inches; if another cask measure 36 inches at the head, and 44 at the bung, the difference is still 8 inches; but the ratio in both cases is not the same. The augment, however, for the two, differs extremely little, as the following examples will show.

584. The formula for the frustum of the prolate spheroid is $\frac{L(2\ B^2 + H^2)}{3\ c}$

For the cylinder it is

$$\frac{L\ D^2}{c};\ \text{or,}\ \frac{L\ 3\ D^2}{3\ c}$$

Consequently, when the two are equal,

$$3\ D^2 = 2\ B^2 + H^2$$

$$D^2 = \frac{2\ B^2 + H^2}{3} = \frac{2\ B^2 + 1\ H^2}{2 + 1}$$

$$= \frac{6\ B^2 + 3\ H^2}{9}$$

$$\therefore D = \tfrac{1}{3}\sqrt{6\ B^2 + 3\ H^2}$$

585. Take the cask with the head 16, bung 24.

$$
\begin{aligned}
D &= \tfrac{1}{3}\sqrt{\overline{6 \times 24^2} + \overline{3 \times 16^2}} \\
&= \tfrac{1}{3}\sqrt{3456 + 768} \\
&= \tfrac{1}{3}\sqrt{4224} \\
&= 21.6 \text{ equicylindric diameter.}
\end{aligned}
$$

Deduct 16. head.

Leaves 5.6 augment for 8 inches.

586. Take the cask with the head 36, bung 44.

$$
\begin{aligned}
D &= \tfrac{1}{3}\sqrt{\overline{6 \times 44^2} + \overline{3 \times 36^2}} \\
&= \tfrac{1}{3}\sqrt{11616 + 3888} \\
&= \tfrac{1}{3}\sqrt{15504} \\
&= 41.5 \text{ equicylindric diameter.}
\end{aligned}
$$

Deduct 36. head.

Leaves 5.5 augment for 8 inches.

587. It will thus be seen that, with a difference of 20 inches in the two casks, there is only $\frac{1}{10}$th of an inch variation in the augment, a discrepancy wholly unimportant, when we regard the difficulty of taking the exact dimensions of a cask to the tenth of an inch, and which is still lessened by the fact, that, with so small a head as 16 inches, the bung is not made to

exceed it by 8 inches. For these reasons it will be evident that the numbers for the SPH$^{\text{D}}$. line will be best computed by taking an average-sized cask as a standard; say with the head 24 inches.

588. The formula is $3\ D^2 = 2\ B^2 + H^2$.

Put $B = H + m$; then

$$\begin{aligned} 2\ B^2 &= 2\ H^2 + 4\ H\ m + 2\ m^2 \\ H^2 &= H^2 \\ \therefore\ 3\ D^2 &= \overline{3\ H^2 + 4\ H\ m + 2\ m^2} \end{aligned}$$

$$D = \sqrt{\frac{3\ H^2 + 4\ H\ m + 2\ m^2}{3}}$$

589. Let the head be 24, and the bung 25; $\therefore\ m = 1$.

$$\begin{array}{rrl} 3\ H^2 = & 1728 & \\ 4\ H\ m = & 96 & \\ 2\ m^2 = & 2 & \\ 3 \,| & \overline{1826} & \\ \sqrt{608.66} = & 24.67 & \text{equicylindric diameter.} \\ & 24. & \text{head.} \\ & \overline{.67} & \text{augment for 1 inch.} \end{array}$$

Set .67 on the SPH$^{\text{D}}$. line over 1 inch.

590. Let the head be 24, and the bung 2 inches more.

$$\begin{aligned} 3\,H^2 &= 1728 \\ 4\,H\,m &= 192 \\ 2\,m^2 &= 8 \\ 3&\,|\overline{1928} \\ \sqrt{642.66} &= 25.35 \text{ equicylindric diameter.} \\ & \ 24. \quad \text{head.} \\ & \ 1.35 \text{ augment for 2 inches.} \end{aligned}$$

Set 1.35 on the SPH^{D} line over 2 inches.

591. Let the head be 24, and the bung 3 inches more.

$$\begin{aligned} 3\,H^2 &= 1728 \\ 4\,H\,m &= 288 \\ 2\,m^2 &= 18 \\ 3&\,|\overline{2034} \\ \sqrt{678} &= 26.04 \text{ equicylindric diameter.} \\ & \ 24. \quad \text{head.} \\ & \ 2.04 \text{ augment for 3 inches.} \end{aligned}$$

Set 2.04 on the SPH^{D} line over 3 inches.

592. These examples will be sufficient for showing the manner of computing a table of augments from 0 to 12 inches, the highest number on the rule. Calculate them for .1 ; .2 ; .3 ; &c. ; and take proportional differences, as for the X lines.

593. The head rod has been described as being fur-

nished with a projecting brass index, or finger piece. This is on the slide; the head rod, in fact, combining in itself a six-foot measure, divided into inches and tenths; a Slide-rule, containing the usual A, B, C, and D lines; a line marked Seg. Ly., for taking the ullage of a lying cask, hereafter to be explained; and the SPH^{D}. line, just described, for reducing a spheroidal cask to a cylinder. Along the top of the face of the rule, on the side containing the index, is laid down the C line. Under this, on the upper part of the slide, is the D line. This formerly *commenced* at 18.78, the gauge point for a cylinder. Within the last few months the line has been made to commence at a lower number; and the gauge point (of course still at 18.78) falls a few inches from the end, where it is marked with a piece of brass. The lower part of the slide, to the right of the index, is a line of inches and tenths, running from left to right; under this, on the stock, is a line of inches and tenths, ending where the other began, and running backwards from right to left. On the slide, to the left of the index, is the SPH^{D}. line; and immediately under that a short line of inches and tenths, running backwards to the commencement of the slide.

594. It has already been seen that the object of the SPH^{D}. line is to add a certain number to the head diameter, for the purpose of reducing the cask to a cylinder. To use the SPH^{D}. line, then, set the face of

the index against the head diameter, taken on the lower part of the stock. Then look to the bung diameter on the same line; and over it, on the SPH^{D}. line, will be a found a number which is to be added to the head diameter, to give the equicylindrical diameter. The short line of inches, however, just under the SPH^{D}. line, saves the trouble of adding this number; for, by selecting the SPH^{D}. number on this short inch-line, immediately under it, on the stock, is found the sum of the head and spheroid number, showing, at once, the equicylindrical diameter. Having obtained this, all that is required is to set the gauge point, 18.78, under the length, taken on the upper or C line; then over the equicylindrical diameter, selected on the upper part of the slide, or D line, will be found the content, as usual, on the C line, according to the formula,

$$\frac{L\,D^2}{18.78^2}$$

EXAMPLES.

595. The length of a mountain butt is 41 inches, the head 26.6, bung 32: required the content.

Bring the face of the index, or finger piece, to

26.6; over 32 is 3.7, SPH^{D}.; under 3.7, inch line, is 30.3, equicylindric diameter.

$$\text{Then} \quad \frac{41 \times 30.3^2}{18.78^2} = ?$$

Set the gauge point under 41; over 30.3 is 107 gallons. (See Question 544.)

596. The length of a Lisbon hogshead is 29 inches, head 25, bung 28, content?

Bring the face of the index to 25; over 28 is 2, SPH^{D}.; under 2, inch line, is 27, equicylindric diameter.

$$\text{Then} \quad \frac{29 \times 27^2}{18.78^2} = ?$$

Set the gauge point under 29; over 27 is 60 gallons. (See Question 545.)

597. The length of a sherry butt is 42.5 inches, head 26.5, bung 32.5, content?

Bring the face of the index to 26.5; over 32.5 is 4.1, SPH^{D}.; under 4.1, inch line, is 30.6, equicylindric diameter.

$$\text{Then} \quad \frac{42.5 \times 30.6^2}{18.78^2} = ?$$

Set the gauge point under 42.5; over 30.6 is 113 gallons.

598. The length of a Teneriffe hogshead is 32 inches, head 19, bung 24.5, content?

Bring the face of the index to 19; over 24.5 is 3.8, SPH^{D}; under 3.8, inch line, is 22.8, equicylindric diameter.

$$\text{Then} \quad \frac{32 \times 22.8^2}{18.78^2} = ?$$

Set the gauge point under 32; over 22.8 is 47 gallons.

599. The length of a Spanish red hogshead is 30 inches, head 21, bung 24, content?

Bring the face of the index to 21; over 24 is 2, SPH^{D}; under 2, inch line, is 23, equicylindric diameter.

$$\text{Then} \quad \frac{30 \times 23^2}{18.78^2} = ?$$

Set the gauge point under 30; over 23 is 45 gallons. (See Question 546.)

600. The length of a Rhenish four aum is 41 inches, head 28, bung 34.8, content?

Bring the face of the index to 28; over 34.8 is 4.7, SPH^{D}; under 4.7, inch line, is 32.7, equicylindric diameter.

$$\text{Then} \quad \frac{41 \times 32.7^2}{18.78^2} = ?$$

Set the gauge point under 41; over 32.7 is 124 gallons.

601. A Charente brandy puncheon is 46 inches long, 26.5 head diameter, 31 bung: required the content.

Bring the index to 26.5; over 31 is 3.1, SPH^{D}.; under 3.1, inch line, is 29.6, equicylindric diameter.

$$\text{Then } \frac{46 \times 29.6^2}{18.78^2} = ?$$

Set the gauge point under 46; over 29.6 is 114 gallons. (See Question 547.)

602. A Bourdeaux brandy hogshead is 37.5 inches long, head diameter 25.5, bung 28.5, content?

Bring the index to 25.5; over 28.5 is 2, SPH^{D}.; under 2, inch line, is 27.5, equicylindric diameter.

$$\text{Then } \frac{37.5 \times 27.5^2}{18.78^2} = ?$$

Set the gauge point under 37.5; over 27.5 is 80 gallons.

603. A Genoa butt is 46.8 inches long, head diameter 28.6, bung 32.5; content?

Bring the index to 28.6; over 32.5 is 2.7, SPH^{D}.; under 2.7, inch line, is 31.3, equicylindric diameter.

$$\text{Then } \frac{46.8 \times 31.3^2}{18.78^2} = ?$$

Set the gauge point under 46.8; over 31.3 is 130 gallons.

604. A Gallipoli butt is 44.6 inches long, head diameter 36.5, bung 41.4, content?

Bring the index to 36.5; over 41.4 is 3.4, SPH^{D}.; under 3.4, inch line, is 39.9, equicylindric diameter.

$$\text{Then} \quad \frac{44.6 \times 39.9^2}{18.78^2} = ?$$

Set the gauge point under 44.6; over 39.9 is 201 gallons.

605. A long breaker is 54 inches in length, head diameter 20.4, bung 24.6, content?

Bring the index to 20.4; over 24.6 is 2.9, SPH^{D}.; under 2.9, inch line, is 23.3, equicylindric diameter.

$$\text{Then} \quad \frac{54 \times 23.3^2}{18.78^2} = ?$$

Set the gauge point under 54; over 23.3 is 83 gallons.

606. A West India rum puncheon is 34.5 inches long, head diameter 27.6, bung 32.3, content?

Bring the index to 27.6; over 32.3 is 3.2, SPH^{D}.; under 3.2, inch line, is 30.8, equicylindric diameter.

$$\text{Then} \quad \frac{34.5 \times 30.8^2}{18.78^2} = ?$$

Set the gauge point under 34.5; over 30.8 is 93 gallons.

607. It is to be understood, that, in all these casks, a deduction has already been made in the length, to allow for the excess in considering them as perfectly spheroidal.

A cask may be known to be a perfect spheroid ermediate diameter, taken by measure, agrees und by the following rule:—

the square root of ¼ the sum of three are of the bung and the square of the is the same, ½ the square root of

$$= \sqrt{\frac{3\,B^2 + 1\,H^2}{3 + 1}}$$

$$+ H^2$$

EXAMPLE.

diameter of a spheroidal cask is 32, the intermediate diameter.

$$= 30.2$$

In the annexed circle take two equal versed sines, r and r; divide the intervening distance into 4 equal parts, and erect 4 perpendiculars to meet the curve; then $4 + 2r =$ the length of the line. Let b denote the bung, h the head, and m the intermediate diameter.

$$\begin{aligned}
\text{Then } h^2 &= (4 + r)r \\
&= 4r + r^2 \\
b^2 &= (2 + r)^2 \\
&= 4 + 4r + r^2 \\
\therefore 3b^2 &= 12 + 12r + 3r^2 \\
h^2 &= \phantom{12 + {}} 4r + r^2 \\
\hline
3b^2 + h^2 &= 12 + 16r + 4r^2 \\
\frac{3b^2 + h^2}{4} &= 3 + 4r + r^2 \\
&= (3 + r)(1 + r) \\
&= m^2 \\
\therefore m &= \sqrt{\frac{3b^2 + h^2}{4}} = \sqrt{\frac{3b^2 + 1h^2}{3 + 1}} \\
&= \tfrac{1}{2}\sqrt{3b^2 + h^2}
\end{aligned}$$

611. Now if, in a cylinder, two sections be taken, one perpendicular to the axis, and the other oblique to it, in such manner that the transverse of the ellipse shall fall over the diameter of the circle, $4 + 2r$; and the semi-conjugate over the semi-diameter, b, in such manner that its extremities shall be equidistant from the extremities of b; then, if an ordinate be taken to fall over h, the intermediate

608. A cask may be known to be a perfect spheroid if its intermediate diameter, taken by measure, agrees with that found by the following rule:—

609. Take the square root of $\frac{1}{4}$ the sum of three times the square of the bung and the square of the head; or, which is the same, $\frac{1}{2}$ the square root of their sum—that is,

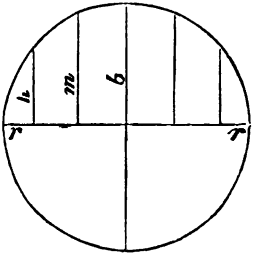
$$\delta = \sqrt{\frac{3\,B^2 + H^2}{4}} = \sqrt{\frac{3\,B^2 + 1\,H^2}{3 + 1}}$$
$$= \tfrac{1}{2}\sqrt{3\,B^2 + H^2}$$

EXAMPLE.

610. The bung diameter of a spheroidal cask is 32, the head 24: required the intermediate diameter.

$$\begin{array}{rl} 3 \times \left.\begin{array}{r} 32 \\ 24 \end{array}\right]^2 & \begin{array}{l} = 3072 \\ = 576 \end{array} \\ & \quad 4\,)\overline{3648} \\ & \quad \sqrt{912} = 30.2 \end{array}$$

DEMONSTRATION.



In the annexed circle take two equal versed sines, r and r; divide the intervening distance into 4 equal parts, and erect 4 perpendiculars to meet the curve; then $4 + 2r$ = the length of the line. Let b denote the bung, h the head, and m the intermediate diameter.

$$\begin{aligned}
\text{Then } h^2 &= (4 + r)r \\
&= 4r + r^2 \\
b^2 &= (2 + r)^2 \\
&= 4 + 4r + r^2 \\
\therefore 3b^2 &= 12 + 12r + 3r^2 \\
h^2 &= \phantom{12 + {}} 4r + r^2 \\
\hline
3b^2 + h^2 &= 12 + 16r + 4r^2 \\
\frac{3b^2 + h^2}{4} &= 3 + 4r + r^2 \\
&= (3 + r)(1 + r) \\
&= m^2 \\
\therefore m &= \sqrt{\frac{3b^2 + h^2}{4}} = \sqrt{\frac{3b^2 + 1h^2}{3 + 1}} \\
&= \tfrac{1}{2}\sqrt{3b^2 + h^2}
\end{aligned}$$

611. Now if, in a cylinder, two sections be taken, one perpendicular to the axis, and the other oblique to it, in such manner that the transverse of the ellipse shall fall over the diameter of the circle, $4 + 2r$; and the semi-conjugate over the semi-diameter, b, in such manner that its extremities shall be equidistant from the extremities of b; then, if an ordinate be taken to fall over h, the intermediate

ordinate will fall over *m*; and the ordinates in the ellipse will be equal to the ordinates in the circle.

612. Required the content of the two following casks, by the rule for frustums, with the intermediate diameter; also by the SPH$^{\text{D}}$. line on the head rod:—

613. The length of a cask is 40 inches, the head diameter 24, bung 32, intermediate 30.2, content?

1st.—By the rule for frustums—

$$\frac{40}{46^2}\left\{\left.\begin{array}{l} \overline{24.} \\ 32. \\ 60.4 \end{array}\right|^2\right. \begin{array}{l} = 10.8 \\ = 19.4 \\ = 69. \\ \hline \ 99.2 \\ \hline \end{array}$$

2nd.—By the spheroid line—

Bring the index to 24; over 32 is 5.6, SPH$^{\text{D}}$.; under 5.6, inch line, is 29.6, equicylindric diameter.

$$\text{Then}\quad \frac{40 \times 29.6^2}{18.78^2} = ?$$

Set the gauge point under 40; over 29.6 is 99.3 gallons.

614. The length of a cask is 40 inches, the head diameter 30, bung 36, intermediate 34.6, content?

By rule for frustums—

$$\frac{40}{46^2}\cdot\left\{\begin{array}{l} \overline{30.\;\;}\big)^2 \\ 36. \\ 69.2 \end{array}\right| \begin{array}{l} = 17. \\ = 24.5 \\ = 90.5 \\ \hline \;\;\; 132. \\ \hline \end{array}$$

By spheroid line—

Bring the index to 30; over 36 is 4.15, SPH$^{\text{D}}$; under 4.15, inch line, is 34.15, equicylindric diameter.

$$\text{Then} \quad \frac{40 \times 34.15^2}{18.78^2} = ?$$

Set the gauge point under 40; over 34.15 is 132 gallons.

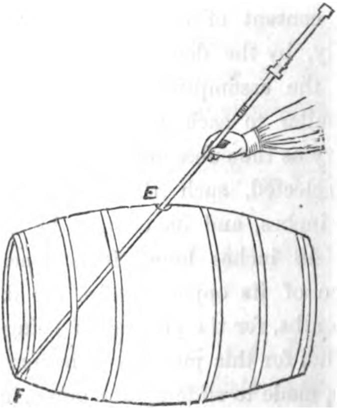
THE DIAGONAL ROD.

615. The content of a cask may also be found, approximately, by the diagonal rod, which is constructed on the assumption that the generality of casks are similar to each other; and, consequently, that they vary as the cubes of their like dimensions. A cask is selected, such that its diagonal shall measure 48 inches, and its content be 250 gallons. The rod is 48 inches long; it is generally made square. Two of its opposite sides are divided into inches and tenths, for the purpose of being used as a bung rod; and, for this purpose, it is furnished with a brass plate, made to slide up and down, in order that

it may be brought over the bung hole, when used for measuring the diameter. The under side of this plate is furnished with two projecting slips of brass, each one inch in length, and divided into 4 equal parts by notches, denoting $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$ of an inch. When it is employed as a diagonal rod, this plate is removed near the top, to be out of the way, as shown in the drawing. At the top of the rod, on one of the two remaining sides, is placed 250 gallons. The position of the other numbers is found by the proportion before mentioned. For instance, suppose the number of gallons to be 144: required the diagonal in inches.

$$250 : 48^3 :: 144 : 40^3, \text{ nearly.}$$

Hence 144 is to be placed against 40 inches; and



so of the rest. The E line of the Author's Compound Slide-rule exhibits the whole of the numbers of the diagonal rod at once.

616. To use this rod, insert it into the bung-hole of the cask, in such manner that the extremity of the rod may meet the head where it intersects the staves of the opposite sides of the cask, as E F, and the content will be exhibited in gallons on the diagonal line, reckoning from the end of the rod, which is tipped with brass, to the centre of the bung-hole; but, as this hole may not be truly in the centre, nor the cask perfectly uniform, the diagonal is to be measured to each end of the cask; and, if the two do not agree, half their sum is to be taken for the content.

617. The remaining side of the rule is for finding the content of the area of a cylinder at the depth of one inch. The number of cylindrical inches in

1 gallon is 353.036 = 18.78^2
2 gallons 706.072 = 26.57
3 ,, 1059.108 = 32.54
4 ,, 1412.144 = 37.57
5 ,, 1765.180 = 42.01
6 ,, 2118.216 = 46.02

Hence 1 gallon is placed at 18.78 inches; 2 at

26.57, &c.; therefore, by applying the rod to the diameter of a cylinder, we obtain its content in gallons, at the depth of 1 inch; consequently, if this area be multiplied by the perpendicular depth, the product will be the content of the vessel. For example, suppose a cylinder 9 inches deep, and 37.57 inches diameter. Applying the rod to take the diameter, we find 4 gallons on the area line; then 4 × 9 = 36 gallons, the content of the cylinder.

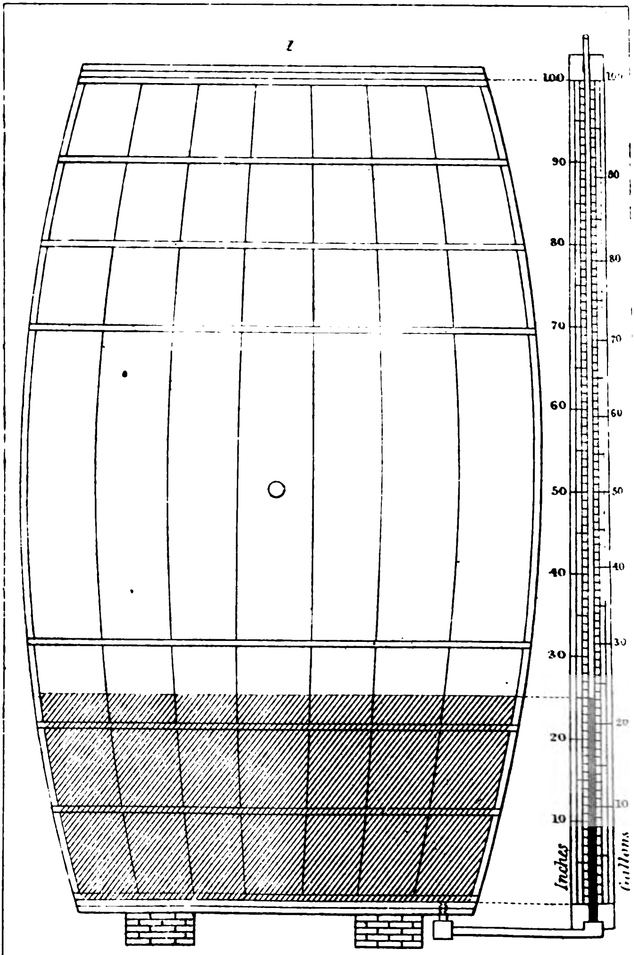
OF ULLAGING CASKS.

618. When a cask is only partly filled it is said to be on ullage; the depth of the liquor is termed the wet ullage; the space above it, the dry ullage. For the purpose of facilitating the finding of this ullage, a spheroidal cask is selected as a standard, from which a line is laid down upon the rule, to serve as a comparison with others. This line may be graduated from computation, or by actual measure. The latter method is the easiest, and is as follows:—

Ascertain the exact quantity the selected cask will hold, and divide it into 100 equal parts. Each of these equal parts, for the sake of distinction, may be called measures.

Into the bottom of the cask, underneath, let there be screwed a brass tube, or tap, closed at the lower

CASK GAGING.

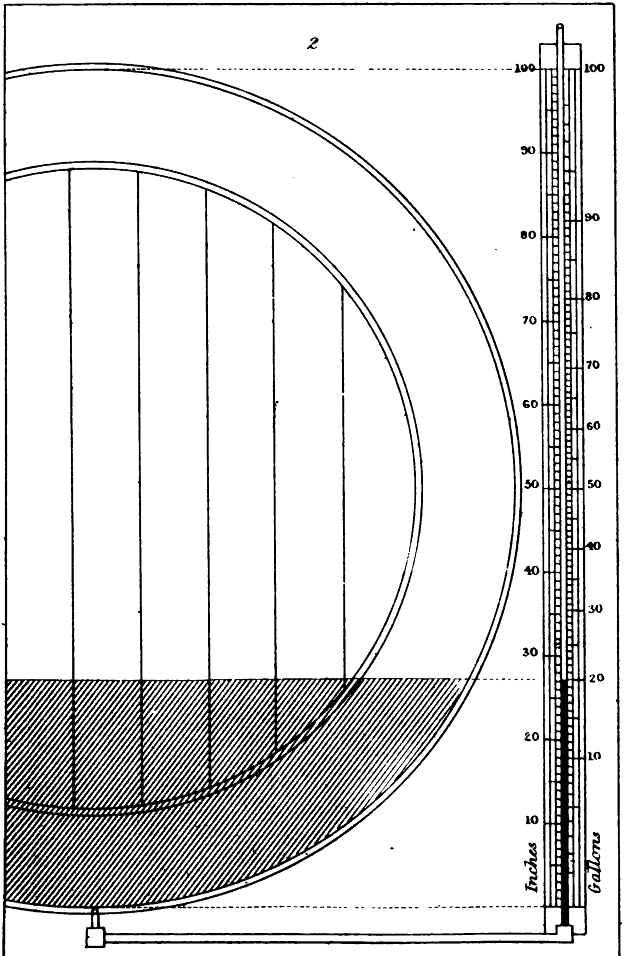


Ullage of Standard Standing Cask.

Pinxit.

ULLAGE SCALES.

2



Ullage of Standard Lying Cask.

end, and perforated on one side with a female or counter screw, for the purpose of receiving an arm, or brass tube, of sufficient length to reach beyond the line falling perpendicularly from the bulge of the cask. The end of this tube is to be turned up at a right angle, with a little cup; and the tap, screwed into the bottom of the cask, is to be of such a length that it shall leave the upper part of the cup somewhat *below the level* of the inside of the cask. Into the cup let there be fixed a glass tube, open at both ends, to stand perpendicularly, and long enough to reach a little higher than the cask. Behind the glass tube is to be fixed a long flat piece of box or other wood, after the manner of a thermometer, to receive graduations.

Then, to construct the ullage line, proceed as follows:—

Pour into the glass tube a little water, for the purpose of finding the level of the inside of the cask, which is indicated by the black mark at the bottom (see Plate, annexed). This being done, mark the height on the wood, to the right of the tube. Pour into the cask 1 measure of water; observe the height to which it rises in the tube; mark it, and number it 1. Pour in another measure; mark, and number it 2; and so proceed till the cask is full, and the number 100 is reached on the scale. This being done, divide the same distance on the left of the tube into 100 *equal* parts. These equal parts may now *be considered* as inches, and the numbers on the right as gallons, as it is easier to refer to them by these names than by any other; and it is simply for reference that they are required. The line on the right of the tube is, of course, the ullage line, and is to be transferred to the Slide-rule by marking the graduations opposite the B or C line, exactly as in the diagram. The figure on the left exhibits the S S, or segment standing line; that on the right, the S L, or segment lying line. Both are accurately drawn, so that computations may be made with them the same as with the Slide-rule.

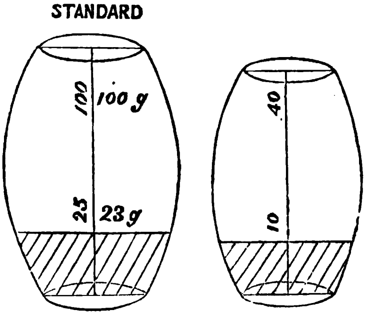
619. The construction of the lines being clearly understood, the mode of using them will be easily comprehended.

620. To find the ullage of a cask.

Measure the depth of the cask, and the depth of the ullage. Also compute the total content of the cask; then proceed as follows:—

EXAMPLES.

621. Suppose the depth of a standing cask is 40 inches, the depth of the wet inches 10; and that the cask holds, when full, 96 gallons; what is the ullage?



As the scales on the diagram are not made to slide, I must work this numerically, and say—

As the total depth of my cask
  Is to the total depth of the standard cask;
So is the partial depth of my cask
  To the partial depth of the standard cask.

That is,

Inches. Inches. Inches. Inches.
40 of mine : 100 standard : : 10 of mine : 25 standard

622. But, inasmuch as I want to find, not the depth, but the content, I refer to the ullage line on the diagram of the standard standing cask, and against 100 inches I find 100 gallons, and against 25 inches 23 gallons; hence the above proportion becomes

Inches. Gallons. Inches. Gallons.
40 of mine : 100 standard : : 10 of mine : 23 standard

623. Having thus ascertained that the ullage of the standard cask for the corresponding depth is 23 gallons, I reverse the order of proceeding, and, beginning with the standard, say—

As the total content of the standard cask
  Is to its partial content;
So is the total content of my cask
  To its partial content.

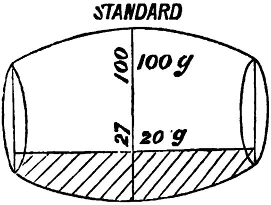
Viz.,

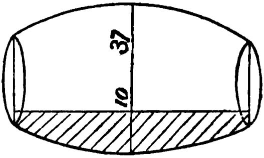
Gallons.		Gallons.		Gallons.		Gallons.
100 Total	:	23 Partial	::	96 Total	:	22 Partial

I thus find the ullage to be 22 gallons.

So with a lying cask.

624. Suppose the bung diameter of a lying cask to be 37 inches, the depth of the wet inches 10, and the cask capable of holding 48 gallons: required the ullage.





As the total depth of my cask
  Is to the total depth of the standard cask;
So is the partial depth of my cask
  To the partial depth of the standard cask.

Inches.		Inches.		Inches.		Inches.
37 of mine	:	100 standard	::	10 of mine	:	27 standard

Then, referring to the diagram for the standard lying cask, against 100 inches is 100 gallons, and against 27 inches 20 gallons; hence the above proportion becomes

Inches.		Gallons.		Inches.		Gallons.
37 of mine	:	100 standard	::	10 of mine	:	20 standard

Then, reversing the order of proceeding, and beginning with the content of the standard, I say—

As the total content of the standard cask
 Is to its partial content;
So is the total content of my cask
 To its partial content.

Viz.,

Gallons.		Gallons.		Gallons.		Gallons.
100 Total	:	20 Partial	::	48 Total	:	9.6 Partial

So 9.6 gallons is the ullage required.

625. Now, if, instead of referring to the diagram, we make use of the Slide-rule; by means of the movable B line we are enabled to avoid the first stating, and to say at once—

As the depth of the given cask on B
 Is to 100 gallons on S;
So are the wet inches on B
 To the standard ullage on S.

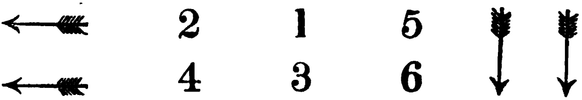
Then, turning to the A and B lines—

As 100 gallons on A
  Are to the standard ullage on B;
So is the content of the given cask on A
  To the required ullage on B.

626. Consequently, if the capacity of the cask be more than 100 gallons, its ullage will be greater than the ullage of the standard cask; if less, less. Also,

627. By multiplying the ullage of the standard cask by the content of your own cask, and striking off two decimals, the required ullage is found without the A and B lines of the Slide-rule.

628. Supposing your cask placed to the right of the standard, the terms of the two statings go in the order 1, 2, 3, 4; 2, 4, 5, 6; as follows:—



Examples of Standing Casks.

629. Required the ullage of a port pipe, containing, when full, 118 gallons, the depth of the cask 50.2 inches, of the wet inches 40.5.

Set the numbers as subjoined, beginning with the depth in the middle, and placing the 100 gallons, the known content of the standard, on the left, thus:—

Gallons. Inches. Gallons.
100 = 50.2 = 118
= 40.5 =

Then say,

Inches. Gallons. Inches. Gallons.
50.2 B : 100 Seg. St. :: 40.5 B : 82.6 Seg. St.

630. Without, however, setting down the stating, simply put the 82.6 under the 100, and the numbers will stand thus:—

Gallons. Inches. Gallons.
100 = 50.2 = 118
82.6 = 40.5 =

Then say,

Gallons. Gallons. Gallons. Gallons.
100 A : 82.6 B :: 118 A : 97.5 B

631. But as these terms, except the last, have already appeared, simply put the 97.5 under the 118, and the numbers will stand thus:—

Gallons. Inches. Gallons.
100 = 50.2 = 118
82.6 = 40.5 = 97.5
Standard Given Cask

So 97.5 gallons is the ullage required.

632. Required the ullage of a Lisbon pipe, containing, when full, 123 gallons, the depth of the cask 47.5 inches, of the wet inches 19.

Gallons. Inches. Gallons.
100 = 47.5 = 123
39 = 19 = 48

Inches. Gallons. Inches. Gallons.
47.5 B : 100 Seg. St. :: 19 B : 39 Seg. St.

Gallons. Gallons. Gallons. Gallons.
100 A : 39 B :: 123 A : 48 B, Ullage.

Examples of Lying Casks.

633. Required the ullage of a Rhenish vat, containing, when full, 255 gallons, the bung diameter 42 inches, the wet inches 20.1.

Gallons. Inches. Gallons.
100 = 42 = 255
47 = 20.1 = 120

Inches. Gallons. Inches. Gallons.
42 B : 100 Seg. Ly. :: 20.1 B : 47 Seg. Ly.

Gallons. Gallons. Gallons. Gallons.
100 A : 47 B :: 255 A : 120 B, Ullage.

634. Required the ullage of a Spanish brandy pipe,

containing, when full, 101 gallons, the bung diameter 28.8 inches, the wet inches 17.

Gallons.		Inches.		Gallons.
100	=	28.8	=	101
63	=	17	=	63.6, Ullage.

Inches.		Gallons.		Inches.		Gallons.
28.8 B	:	100 Seg. Ly.	::	17 B	:	63 Seg. Ly.

Gallons.		Gallons.		Gallons.		Gallons.
100 A	:	63 B	::	101 A	:	63.6 B, Ullage.

635. The terms of the first stating, it will be observed, are alternately inches and gallons. Of late years, since the name of Proportion has come to be substituted for that of the Rule of Three, (which expressions are not necessarily identical,) much discussion has taken place with respect to homogeneous and heterogeneous stating, it being maintained, that to say 2 *oz* : 3 *s* :: 4 *oz* : 6 *s* is an absurdity, and that it should be 2 *oz* : 4 *oz* :: 3 *s* : 6 *s*, because there can be no ratio between such incongruous things as ounces and shillings. Of course not; all that is meant is, that there is the same *commercial* relation between 2 *oz* and 3 *s* as there is between 4 *oz* and 6 *s*, and that the stating 2 *oz* : 3 *s* :: 4 *oz* : 6 *s* should be read—As 2 ounces are *worth* 3 shillings, so are 4 ounces *worth* 6 shillings. The contest seems to resolve itself into a dispute about the meaning of a

couple of dots. But, however it may be decided for numerical computation, when we come to the Slide-rule we have no alternative, the principle of the instrument being always the golden Rule of Three, presenting *a series* of antecedents and consequents, the former being of the same name with one another, and the latter the same; so that we ***must*** say 2 *oz* : 3 *s* :: 4 *oz* : 6 *s* :: 6 *oz* : 9 *s* :: 8 *oz* : 12 *s*, &c. If the terms happen to be homogeneous, it is, at the same time, strictly Proportion; if they are heterogeneous, it is the Rule of Three; and the ratio, if it is necessary to insist upon ratio, and any advantage can be shown to be derived from so considering it, exists in the abstract numbers, not in the things themselves. With the ullage lines, however, the ratio exists, not between the gallons, the measures of quantity, but between the inches, the measures of extension, which those quantities represent.

636. Should it be desired to describe, on a large scale, an ellipse representing the section of a spheroidal cask, of which the head and bung diameters are given, the length of the transverse, which is necessary for the purpose, may be found as follows:—

637. Suppose the ellipse described, and the required lines drawn within it. Mark half the bung diameter as b, half each head diameter as h, each distance between the bung and head as l, and each versed sine as m. Then

$$(l + m)^2 : b^2 :: (2\,l + m)m : h^2$$

$$(l^2 + 2\,l\,m + m^2)h^2 = (2\,l\,m + m^2)b^2$$

$$l^2\,h^2 + 2\,l\,m\,h^2 + m^2\,h^2 = 2\,l\,m\,b^2 + m^2\,b^2$$

$$l^2\,h^2 = (m^2 + 2\,l\,m)(b^2 - h^2)$$

$$m^2 + 2\,l\,m = \frac{l^2\,h^2}{b^2 - h^2}$$

$$m^2 + 2\,l\,m + l^2 = \frac{l^2\,h^2}{b^2 - h^2} + l^2$$

$$= \frac{l^2\,b^2}{b^2 - h^2}$$

$$m + l = \sqrt{\frac{l^2\,b^2}{b^2 - h^2}}$$

$$m = l\,b\sqrt{\frac{1}{b^2 - h^2}} - l$$

$$= l\left\{b\sqrt{\frac{1}{(b + h)(b - h)}} - 1\right\}$$

638. Suppose H, the head diameter of a spheroidal cask, to be 24 inches; B, the bung diameter, 32; and L, the length, 40.4; then $h = 12$; $b = 16$; $l = 20.2$.

$$\therefore\ m = 20.2\left\{16\sqrt{\frac{1}{28 \times 4}} - 1\right\}$$

$$= 20.2\left\{16\sqrt{\frac{1}{112}} - 1\right\}$$

$$= 20.2\left\{16\sqrt{\frac{7}{784}} - 1\right\}$$

$$= 20.2\left\{\frac{4}{7}\sqrt{7} - 1\right\}$$

$$\begin{array}{rl}
\sqrt{7} = 2.64575 & \\
4 & \\
\hline
7 \,|\, 10.583 & \\
\hline
1.512 & \\
1. & \\
\hline
.512 & \\
20.2 & \\
\hline
1024 & \\
1024 & \\
\hline
10.3424 & = m \\
20.2 & = l \\
\hline
30.5424 & \\
2 & \\
\hline
61.0848 & = \text{transverse.} \\
\hline
\end{array}$$

$$\left.\begin{array}{l} \overline{30.54} \\ 16. \end{array}\right|^{2} \begin{array}{l} = 932.6916 \\ = 256. \end{array}$$

$\sqrt{676.6916} = 26$, the distance of the focus from the centre of the ellipse.

639. By describing on the floor, or any smooth boards, an ellipse of the full size, and drawing the bung and head diameters in the proper position, the reader will obtain a correct idea of a spheroidal cask.

640. The content of the prolate spheroid of these dimensions will be found to be 118 gallons; the content of the middle frustum of the same 100 gallons; half the difference 9, the content of each terminal segment. Then, if we suppose the cask to be filled to $\frac{1}{10}$ of its depth, the fresh segment will be 14.38 inches deep, and its content 17.2 gallons, from which,

deducting the smaller segment, there will be left 8.2 gallons to be placed against 10 inches on the S S line; and in the same manner may any other number be computed. This cask will be found to agree, as nearly as possible, with the S S line of the Slide-rule.

641. The content of particular depths may also be ascertained by calculating intermediate diameters, and computing the capacity by the rule for frustums.

642. Conceiving the bottom half of the cask to be crossed by 8 equidistant diameters, their measures will be as follow:—

$$\sqrt{\begin{array}{rl} 1024 & \\ 1017 & \\ 996 & \\ 975 & \\ 912 & m' \\ 891 & \\ 828 & m'' \\ 765 & m''' \\ 576 & \end{array}}$$

$$\begin{array}{rl} \text{For}\ \ 1024 & \\ 3 & \\ \hline 3072 & \\ 576 & \\ \hline 4|3648 & \\ \hline \sqrt{\ }912 & = m' \\ 3 & \\ \hline 2736 & \\ 576 & \\ \hline 4|3312 & \\ \hline \sqrt{\ }828 & = m'' \\ 3 & \\ \hline 2484 & \\ 576 & \\ \hline 4|3060 & = m''' \\ \hline \sqrt{\ }765 & \end{array}$$

Where m', m'', m''', denote the intermediate diameters.

Then, to find the content of the frustum, of which the lower three numbers are the top, intermediate, and bottom diameters,

$$\begin{array}{rrl} & 828 & \\ 4 \times 765 = & 3060 & \\ & 576 & \\ \cline{2-2} & 4464 & \times 5.05 \times .0004721 = 10.64 \end{array}$$

gallons for $\frac{1}{8}$ of the depth of the cask—that is, for 12.5 inches in 100.

$$\begin{array}{rrl} & 912 & \\ 4 \times 828 = & 3312 & \\ & 576 & \\ \cline{2-2} & 4800 & \times 10.1 \times .0004721 = 22.88 \end{array}$$

gallons for $\frac{1}{4}$ of the depth of the cask—that is, for 25 inches in 100.

$$\begin{array}{rrl} & 996 & \\ 4 \times 891 = & 3564 & \\ & 576 & \\ \cline{2-2} & 5136 & \times 15.15 \times .0004721 = 36.74 \end{array}$$

gallons for $\frac{3}{8}$ of the depth of the cask—that is, for 37.5 inches in 100.

These will be found to agree with the S S line of the Slide-rule.

643. The transverse diameter of the spheroid being known, ordinates may be calculated for any distance from the centre. The plan, however, I have suggested with the glass tube is a far preferable mode of

graduating the contents, especially with a lying cask, the computations for which would be wearisomely tedious, and, after all, a mere matter of curiosity. It is sufficient to know *how* the ullage lines are constructed, to understand fully their application; their graduation may be left to the makers.

---

644. In section 537 are given the calculations for the gauge points for the 1st and 2nd variety X lines; it was not at the time deemed necessary to insert any others; but, as several Slide-rules contain X lines for Hutton's and Young's varieties also, the calculations for every fifth graduation are here subjoined. The intervening ones can be interpolated by hand, by comparing them with a portion of the A line having the same interval, containing the like number of subdivisions.

Hutton's variety.

$$L\left(39\,B^2 + 26\,H\,B + 25\,H^2\right)\frac{1}{31773.24}$$

$39 + 26.\overline{H \div B} + 25.\overline{H \div B}|^2$

39 + 26 × .5	+ 25 × .25	= 58.25	$\sqrt{31773.24 \div \text{each preceding}}$	= 23.36	over .5
.55	.3025	60.8625		22.85	.55
.6	.36	63.6		22.35	.6
.65	.4225	66.4625		21.87	.65
.7	.49	69.45		21.39	.7
.75	.5625	72.5625		20.92	.75
.8	.64	75.8		20.47	.8
.85	.7225	79.1625		20.03	.85
.9	.81	82.65		19.60	.9
.95	.9025	86.2625		19.19	.95

Young's variety.

$$L(7\,B^2 + 5\,B\,H + 3\,H^2)\frac{1}{5295.54}$$

$$7 + 5.\overline{H \div B} + 3.\overline{H \div B}|^2$$

$7 + 5 \times .5$	$+ 3 \times .25$	$= 10.25$	5295.54 ÷ each preceding	$= 22.73$ over	.5
.55	.3025	10.6575		22.29	.55
.6	.36	11.08		21.86	.6
.65	.4225	11.5175		21.44	.65
.7	.49	11.97		21.03	.7
.75	.5625	12.4375		20.63	.75
.8	.64	12.92		20.24	.8
.85	.7225	13.4175		19.87	.85
.9	.81	13.93		19.51	.9
.95	.9025	14.4575		19.14	.95

645. The following table exhibits the usual forms and dimensions of casks, with the allowance to be made for reducing them to a spheroidal form. The more nearly their shape appears to be of that form, the less, of course, is to be the deduction.

Name of Casks.	Usual allowance upon the length.	Usual allowance upon the bung.	Average Dimensions.			
WINE.			Length	Head.	Bung.	Conten
Port Pipes, stout, well made...	.3 to .5	1.	50.2	22.7	31.5	118
„ Hogsheads, thin, well made	.0	.75	32.	22.8	27.	59
Lisbon Pipes, bad figure	1. to 1.5	1.	47.5	24.8	32.7	123
„ Hogsheads, well made, thin....................	.0	.75	29	25	28	60
Madeira Pipes, stout, ill made	1. to 2.	various	47.5	21.4	28.5	93
„ Hogsheads, do., do....	.3 to .5	1.	32.5	19	23.5	45
Sherry Butts, thin, good figure	.2 to .6	1.	42.5	26.5	32.5	113
„ Hogsheads, thin, well formed	.0	.75	30.5	23	26.5	55
Mountain Butts, well made ...	.4 to .6	1.	41.	26.6	33.	107
Teneriffe Pipes, bad figure ...	.5 to 1.2	1.	47.4	23.3	28.8	98
„ Hogsheads, thin, long	.2 to .3	.75	32	19	24.5	47
Fayal Pipes, various	.4 to .6	1.	47.5	21.7	27.7	89
Sicilian Pipes, stout made......	.6 to 1.	1.	46.4	22.4	28.6	93
„ Hogsheads, stout, ill made	.4 to .6	1.	35.6	17.	23.1	45
Cape Pipes, various...............	.8 to 1.	1.	47.8	22.8	28.5	96
„ Hogsheads, stout made...	.2 to .4	1.	31.5	19.4	23.6	44
Spanish Red, or Catalonia Pipes	.4 to .7	1.	47.2	25.6	28.8	103

Name of Casks.	Usual allowance upon the length.	Usual allowance upon the bung.	Average Dimensions.			
			Length	Head.	Bung.	Content
WINE (continued).						
Spanish Red, or Catalonia Pipes, cocked bung.........	.3 to 6.	1.	50.	22.8	31.3	116
Spanish Red Hogsheads.........	.2 to .4	1.	30.	21.	24.	45
Rhenish Vats, good figure......	1.8 to 3.	various	56	36	42	255
„ Four Aums, well made	.6 to .8	1.	41	28	34.8	124
„ Double Aums, good figure.........................	.0 to .2	1.	31.5	23.7	27.5	61
Claret & Burgundy Hogsheads	.0	1.	28.5	22.5	25	47
Figuera Pipes	.3 to .7	1.	50.2	22.7	31.5	118
„ Hogsheads	.0	.75	32	22.8	27	59
BRANDY.						
Charente Puncheons	.2 to .4	various	46.	26.5	31.	114
„ Hogsheads, good fig.	.0 to .5	.75	34.1	22.5	25.9	59
Bourdeaux Puncheons	.4 to .7	1.	46.8	26.	30.5	112
„ Hogsheads, ordinary	.3 to .5	1 to 1.5	37.5	25.5	28.5	80
Spanish Butts, rough	.7 to 1.	1.	43	38	42	202
„ Pipes, ordinary	.4 to .7	1.	46	25.8	28.8	101
Naples Pipes, bad figure	.8 to 1.	1.	46	26.5	31.5	116
GENEVA						
Puncheons, indented staves ...	.4 to .8	1.	44.6	25	29.4	99
Hogsheads, rough	.0	1.	33	19.5	25.5	52
RUM.						
English Puncheons, good fig.	.2 to .4	1.	34.5	27.6	32.3	93
West India, bad figure	.3 to .6	various				
Hogsheads, long thin, Porter Casks	.2 to .4	.75	31	22	26	53
WHISKEY.						
In Puncheons	—	—	—	—	—	—
Also in Wine Pipes and Rum Puncheons.....................	—	—	—	—	—	—
OLIVE OIL.						
Genoa Butts, stout	1.	1.	46.8	28.6	32.5	130
„ Pipes	.6 to .8	1.	44.6	26.5	29.5	102
„ Half Butts	.2 to .4	1.	36.5	23.6	27.6	71
Gallipoli Butts, thick	1. to 1.2	1.	44.6	36.5	41.4	201
„ large Pipes, stout ...	1.	1.	48.6	28.4	33.4	139
„ small Pipes, thin......	.6 to .8	1.	46.6	25.6	29.6	106
„ short Puncheons......	.5	1.	35.6	29.2	32.6	100
„ short Casks, thin ...	.0	1.	29.6	24.6	28.6	62
„ Hogsheads, thin......	.2 to .4	.75	31.4	21.6	26.	53
WHALE OIL.						
Large Butts, stout, well made	2.5	1.2	58.5	40.6	46.6	332
Small Butts, „	2 to 2.2	1.	54	34	40	223
Long Pipes, well made	1.5 to 1.8	1.	54	24.4	29.4	118
Pipes, well made..................	1.	1.	46	24.6	28.8	98
Long Breakers, largest, well made	1.5 to 1.8	1.	54	20.4	24.6	83
Long Breakers, large	1 to 1.2	1.	53	16.4	20.4	55
„ medium	.6 to 1.	1.	45.6	15.6	18.6	40
„ small............	.4 to .6	.75	33.8	13.6	16.	22

MALT GAUGING.

646. In order to convert barley into malt, it is put into a cistern, and covered with water to the depth of 6 inches. Here it must be left till it is sufficiently saturated, which, by law, cannot be less than 40 hours; but the precise time it should remain depends upon the state of the atmosphere and the quality of the grain. As soon as it is deemed sufficiently steeped, the water is let off; and, after remaining to drain for 20 or 30 minutes, the grain is thrown out of the cistern, with large wooden shovels, into the couch frame, which must be of a rectangular form, with upright sides, three being permanent and one movable. The movable side, if formed of planks, must be at least 2 inches thick, and so supported that they shall not bend outward by the pressure of the grain. The couch frame is not allowed to be filled to a greater depth than 30 inches, nor the grain to be in any way compressed but by its own weight. After remaining the proper time in the couch, which must not be less than 26 hours, the grain is taken out, spread upon the floor, and turned from time to time with a shovel. After remaining on the floor for 6 days, it is allowed to be sprinkled, provided it has been kept under water [illegible] cistern for 50 hours. Unless this has

been done, it must remain 12 days before the sprinkling is allowed. After the germination has sufficiently proceeded, farther growth is checked, and the temperature reduced, by spreading it thinner and thinner. It is then dried upon a kiln of perforated tiles, or iron plates, or between cylinders of strong wire work. For drying pale malt it is laid thick upon the kiln, and the fire is applied more gradually than for brown malt, the colour of which is imparted by a strong fire towards the end of the process. Porter malt is spread very thin upon the kiln, generally not deeper than an inch and a half, and dried at a higher temperature, for the purpose of converting the sugar, gum, and starch of the malt into caramel, which, being a powerful colouring agent, imparts to the beverage the peculiar brownness so well known; it also communicates the agreeable bitter flavour, equally familiar, and which is perceptible in the taste of the crust of bread when over-baked. The fire is likewise supplied with faggots of wood, for the purpose of impregnating the malt with the fumes of pyroligneous acid, which impart to it a peculiar acidity not to be obtained by other means.

647. By judicious malting, under favourable circumstances, the bulk is increased by from 3 to 10 per cent.; but this is partly the result of skill and experience, and partly dependent on the quality of

the grain. The Legislature charges the duty upon the assumption that the quantity of malt eventually obtained equals the barley originally wetted; and, to prevent any fraudulent practices, the grain is gauged, from time to time, throughout the whole process of malting; first in the cistern, secondly in the couch, then on the floors, and lastly on the kiln; and deductions are made, according to a settled allowance, for the swell it acquires by absorption of water and germination of the seed. Formerly the allowance upon the grain in the cistern or couch was 20 per cent. In 1830 this was lessened to 17½, which giving much dissatisfaction, an Act was passed in the first year of the reign of her present Majesty, fixing the legal allowance upon the cistern or couch gauge at 18½ per cent. There were, also, formerly two rates of allowance for the grain when germinating on the floor, viz. ⅓ of the gauge if taken before the expiration of 72 hours after being thrown out of the cistern, and one half upon all gauges taken subsequently, whether on floor or kiln. By the Act just mentioned, the allowance on the floor was changed to only one rate, viz. one half, irrespective of any time out of cistern after it had been removed from couch to floor. The enactment of the Legislature, therefore, now in force, is, that from every 100 bushels. gauged in the couch or cistern, 18½ shall be deducted;

and from any gauge taken on the floor, one half. Consequently,

163 floor bushels = 100 couch, or cistern = 81.5 net.

$$\text{Now} \quad \frac{81.5}{163} = \frac{\text{Net}}{\text{Floor}} = .5$$

$$\frac{81.5}{100} = \frac{\text{Net}}{\text{Couch or Cistern}} = .815$$

$$\frac{163}{100} = \frac{\text{Floor}}{\text{Couch or Cistern}} = 1.63$$

Hence, to reduce floor bushels to net, multiply by .5.

To reduce couch or cistern to net, multiply by .815.

To reduce couch or cistern to floor, multiply by 1.63.

648. The duty is always charged upon the highest gauge of the cistern, couch, or floor; and as it is but seldom that the grain expands to more than twice its original bulk, the highest gauge is generally from the couch. The propriety, therefore, of making it rectangular, with the bottom even, and only so slightly inclined as just to drain it, is obvious, as it allows the greatest facility for mensuration. It is, in fact, a fixed measure; the length and breadth, and, consequently, the area, of which, are known; so that nothing farther is required than simply to multiply it by the depth of the grain to obtain at once the entire

content. The grain is levelled as nearly as possible, but as this cannot be done with perfect correctness, the depth is to be ascertained by measuring it at several places, particularly where deepest and shallowest; then, adding all the depths together, and dividing the sum by the number of them for a mean depth. In practice 10 depths are most convenient, as, after adding them together, they require only the decimal point to be removed one figure to the right for obtaining the mean depth.

649. The rules for finding the contents of vessels are the same as those already given for the various solids, observing to use the divisors, factors, and gauge points, for bushels. The gauge points, however, are seldom required, as there is an M D line on the Slide-rule, which answers the purpose more conveniently. This M D, Malt, Depth, inferentially Bushels, Depth—malt being always measured by bushels, and the words therefore taken as synonymous —is an inverted A line, so placed as to commence at 2218.192, the number of cubic inches in a bushel; consequently, from what has been explained before, this number acts as a constant divisor; and therefore we have only over the depth, or one of the dimensions, taken on this line, to set a second dimension on the slide, and to select the third on the A line, when under it will be found the content at once. This line is equally available for gallons, by taking 8 times one

of the dimensions, and should in all cases be employed when the length, breadth, and thickness differ from each other, for the purpose of avoiding the finding of a mean proportional, an operation tedious in itself, and almost always involving the inconvenience of decimals. When two or more numbers have to be squared, as in the frustums of cones and pyramids, the D line is preferable. These, however, though subjoined for the purpose of showing the applicability of the rule, are rather theoretical than practical; but their introduction will exhibit the value of the M D line for other purposes than malt gauging.

650. As there is some little difficulty at first in estimating the values with the M D line, the following observations may be attended to:—

First, the formula is $C = \frac{L\,B\,D}{2218.192}$, the denominator, or constant divisor, being determined by the position of the line; this divisor, it will be observed, contains four integers. Also, the beginning of the M D line is on the right hand; of the B and A lines, on the left hand; hence the following rule:—Observe the number of figures expressing the value of the *commencement* of each of the three lines; add them together, and deduct 4 from the sum; the remainder shows *the number of integers in the first radius of the slide.* The following examples will elucidate the subject:—2218.192 is denoted by V, which simply implies that the M D line is to be used. Any one of the dimensions may be selected on the M D line; but in practice it is best to take the least of the three numbers on it, which, generally, is the depth. The numbers are taken upon the lines in the order denoted by the letters M D, B, A, B.

Example First.

$$\frac{3 \times 80 \times 70}{V} = ?$$

3 M D : 80 B :: 70 A : 7.57 B

Here the commencement of M D contains *one* integer; of B, *two*; of A, *two*. 1 and 2 are 3, and 2 are 5; and 4 from 5 leaves 1. The answer, or quotient, falls in the first radius of the slide; it therefore contains one whole number, and is 7.57.

Example Second.

$$\frac{20 \times 300 \times 30}{V} = ?$$

20 M D : 300 B :: 30 A : 81 B

Commencement of M D, *one* integer; of B, *three*; of A, *two*. 1 and 3 are 4, and 2 are 6; 4 from 6 leaves 2. The answer is therefore 81.

EXAMPLE THIRD.

$$\frac{70 \times 6 \times 5}{V} = ?$$

5 M D : 70 B :: 6 A : .94 B

Of M D, *one*; of B, *two*; of A, *one*. 1 and 2 are 3, and 1 are 4; 4 from 4 = 0. The quotient, therefore, has *no* integers, and is .94.

EXAMPLE FOURTH.

$$\frac{9 \times 3 \times 4.2}{V} = ?$$

9 M D : 3 B :: 4.2 A : .051 B

Of M D, *one*; of B, *one*; of A, *one*. 1 and 1 are 2, and 1 are 3; and 3 — 4 = — 1. The quotient will therefore be a decimal beginning with *one cipher*, viz., .051.

EXAMPLES IN MALT GAUGING.

651. A cistern is 96 inches by 80, and is filled to the depth of 20 inches: required the area and the content.

$$96 \times 80 \left\{ \begin{matrix} \div 2218.192 \\ \times .00045081 \end{matrix} \right\} = 3.4622 \text{ bushels area;}$$

and $3.4622 \times 20 = 69.24$ bushels content.

By the slide-rule—

20 M D : 96 B :: 80 A : 69¼ B

652. There is, however, a ready way of finding the content, which may be explained as follows:—

Cubic inches in a bushel .	2218.192
One half . . .	1109.096
Cut off 3 decimals .	1.109096
Deduct $\frac{1}{10}$th . .	.1109096
Leaves . . .	.9981864
Add $\frac{1}{550}$th . .	.001814
Gives . . .	1.000000 bushel.

Which process is therefore equivalent to dividing by 2218.192.

Hence the following rule:—

653. Multiply the product of two of the dimensions by half the third, cut off three decimals, and

deduct one-tenth; the number left is the quantity of bushels required, very nearly.

If it be desired to be *exact*, for every 550 bushels add 1 bushel, or for every 11 bushels, .02.

Take the previous example—

$$\begin{array}{r} 96 \\ 80 \\ \hline 7680 \\ 10 \\ \hline 76.800 \\ 7.68 \\ \hline 69.12 \text{ bushels.} \\ \hline \end{array}$$

This is sufficiently near for all practical purposes; were it required to be exact, we should say 11 in 69 goes 6, and 6 times .02 = .12 to be added; then

$$\begin{array}{r} 69.12 \\ .12 \\ \hline 69.24 \text{ exact content.} \\ \hline \end{array}$$

654. The length of a floor of malt is 329 inches, the breadth 242, and the depth 5.6: required the content.

$$329 \times 242 \times 5.6 \left\{ \begin{array}{l} \div\ 2218.192 \\ \times\ .000450817 \end{array} \right\} = 201 \text{ bushels.}$$

Or thus,

$$\begin{array}{r} 329 \\ 121 \\ \hline 329 \\ 3948 \\ \hline 39809 \\ 5.6 \\ \hline 238854 \\ 199045 \\ \hline 222.9304 \\ 22.29 \\ \hline 200.64 \\ \hline \end{array} \text{ bushels.}$$

And

$$\begin{array}{r} 11|200 \\ \hline 18 \\ .02 \\ \hline .36 \\ 200.64 \\ \hline 201. \\ \hline \end{array} \text{ exact content.}$$

655. In practice, the content of small floors may be determined by the Slide-rule; but when the floor exceeds 200 bushels, it cannot be so accurately ascertained by the rule. Floor charges, whether large or small, should always be cast by the pen.

656. The length of a rectangular floor of malt is 1900 inches by 600: required the area.

$$1900 \times 600 \left\{ \begin{array}{l} \div\ 2218.192 \\ \times\ .000450817 \end{array} \right\} = 513.93 \text{ bushels.}$$

Or,

$$\begin{array}{r} 1900 \\ 300 \\ \hline 570.000 \\ 57. \\ \hline 513. \\ \hline \end{array} \text{ bushels.}$$

And

$$\begin{array}{r} 11|513 \\ \hline 46\tfrac{1}{2} \\ .02 \\ \hline .93 \\ 513. \\ \hline 513.93 \\ \hline \end{array} \text{ exact content.}$$

657. A couch frame is 424 inches long, 102 broad,

and is filled to the depth of 18 inches: required the content.

$$424 \times 102 \times 18 \left\{ \begin{matrix} \div 2218.192 \\ \\ \times .000450817 \end{matrix} \right\} = 350.94 \text{ bushels.}$$

Or,
$$\begin{array}{r} 424 \\ 102 \\ \hline 848 \\ 424 \\ \hline 43248 \\ 9 \\ \hline 389.232 \\ 38.92 \\ \hline 350.31 \text{ bushels.} \\ \hline \end{array}$$

And
$$\begin{array}{r|l} 11 & 350 \\ \hline & 31\tfrac{1}{2} \\ & .02 \\ \hline & .63 \\ & 350.31 \\ \hline & 350.94 \text{ exact content.} \\ \hline \end{array}$$

658. A heap of malt thrown up into the form of a prismoid is 100 inches by 80 at the base, 70 by 60 at the top, and the depth is 9 inches: required its content?

$$\begin{array}{lrcrcr} \text{Top} & 70 & \times & 60 & = & 4200 \\ \text{Bottom} & 100 & \times & 80 & = & 8000 \\ \text{Sum} & 170 & \times & 140 & = & 23800 \\ \hline & & & & & 36000 \end{array}$$

$$36000 \times 9 \left\{ \begin{matrix} \div 13309.15 \\ \\ \times .00007513 \end{matrix} \right\} = 24.34 \text{ bushels.}$$

The half of 36000 is 18000; dividing this by 6, agreeably to the rule for frustums, and cutting off three decimals, we have 3 left; then, by the rule just explained,

$$\begin{array}{r} 3 \\ 9 \\ \hline 27 \\ 2.7 \\ \hline 24.3 \end{array} \text{ content.}$$

By the slide-rule, taking $\frac{1}{6}$ of the depth,

$$\begin{array}{lrcrcrcr} & \text{M D} & & \text{B} & & \text{A} & & \text{B} \\ \text{Top} & 70 & : & 60 & :: & 1.5 & : & 2.83 \\ \text{Bottom} & 100 & : & 80 & & & : & 5.4 \\ \text{Sum} & 170 & : & 140 & & & : & 16.07 \\ \cline{8-8} & & & & & & & 24.3 \\ \cline{8-8} \end{array}$$

If the depth should not be divisible by 6, the whole depth may be used, and the quantity found be divided by 6 at the end, as in the following example:—

659. A heap of malt is piled up into the form of the frustum of a square pyramid; the side of the base is 50 inches, of the top 40, and the depth is 8 inches: required the content.

$$\begin{array}{ll} \text{Top} & 40 \\ \text{Bottom} & 50 \\ \text{Sum} & 90 \end{array}\left.\vphantom{\begin{array}{l}1\\1\\1\end{array}}\right|^{2} \begin{array}{l} = 1600 \\ = 2500 \\ = 8100 \end{array}$$

$$12200 \times 8 \left\{ \begin{array}{l} \div\ 13309.15 \\ \times\ .00007513 \end{array} \right\} = 7.33 \text{ bushels.}$$

$$\begin{array}{lr}
\text{Or,} & 12200 \\
 & \underline{4} \\
 & 48.800 \\
 & \underline{4.88} \\
 & 6\,|\,\overline{43.92} \\
 & \underline{7.32}
\end{array}$$

By the slide-rule—

$$\begin{array}{lccccccc}
 & \text{M D} & & \text{B} & & \text{A} & & \text{B} \\
\text{Top} & 40 & : & 40 & :: & 8 & : & 5.78 \\
\text{Bottom} & 50 & : & 50 & & & : & 9. \\
\text{Sum} & 90 & : & 90 & & & : & 29.2 \\
 & & & & & & & 6\,|\,\overline{43.98} \\
 & & & & & & & \underline{7.33}
\end{array}$$

Or, by the D line—

$$\frac{8}{115.36^2}\left\{\begin{array}{c} 40 \\ 50 \\ 90 \end{array}\right]^2 \begin{array}{l} = .96 \\ = 1.5 \\ = \underline{4.87} \\ 7.33 \end{array}$$

660. A heap of malt piled against a wall assumes the form of a wedge, its height being 20 inches, the base 60 inches by 11, the length of the top 48 inches: required the content.

$$\begin{array}{lrl}
\text{Top} & 48 \times 0 & \\
\text{Bottom} & 60 \times 11 = & 660 \\
\text{Sum} & 108 \times 11 = & 1188 \\
 & & \overline{1848} \times 20 \left\{\begin{array}{l} \div\ 13309.15 \\ \times\ .00007513 \end{array}\right\} = 2.77 \text{ bushels.}
\end{array}$$

	M D	B		A		B
Top		0	::	8		
Bottom	20 :	11	::	10	:	.99
Sum		: 11	::	18	:	1.78
						2.77

661. A cylindrical vessel of malt is 8 inches deep, the diameter is 42 inches: required the content.

$$42^2 \times 8 \left\{ \begin{array}{l} \div 2824.29 \\ \times .00035407 \end{array} \right\} = 4.9966 \text{ bushels.}$$

By the slide-rule—

42 M D : 42 B : 8 A : 6.36 B, content of the circumscribing square prism.

Now a circle is $\frac{.7854}{1} = \frac{11}{14}$ of its circumscribing square; consequently, the cylinder is $\frac{11}{14}$ of the prism; hence

14 A : 11 B :: 6.36 A : 5 B

Or, preferably, by the D line at once—

$$\frac{8}{53.14^2}\left\{ 42^2 = ? \right.$$

53.14 D : 8 C :: 42 D : 5 C

The gauge point, 53.14, will be found marked with a [illegible] and the letters, M R, Malt, Round—that is, Bushels, Round; the content in bushels; the vessel round, or cylindrical.

662. A vessel in the form of an elliptic cylinder is 48 inches by 36 inches, the depth 10 inches: required the content.

$$48 \times 36 \times 10 \left\{ \begin{array}{l} \div\ 2824.29 \\ \\ \times\ .00035407 \end{array} \right\} = 6.118 \text{ bushels.}$$

By the slide-rule—

48 M D : 36 B :: 10 A : 7.8 B

And

14 A : 11 B :: 7.8 A : 6.1 B

663. A vessel in the form of a cone is 68 inches in diameter at top, and 40 inches deep: required the content in bushels.

$$68^2 \times 40 \left\{ \begin{array}{l} \div\ 8472.87 \\ \\ \times\ .00011802 \end{array} \right\} = 21.8 \text{ bushels.}$$

By the slide-rule—

68 M D : 68 B :: 40 A : 83.2 B

The cylinder is $\frac{11}{14}$ of the prism, and the cone $\frac{1}{3}$ of the cylinder; consequently, the cone is $\frac{11}{14} \times \frac{1}{3} = \frac{11}{42}$ of the prism; hence

42 A : 11 B :: 83.2 A : 21.8 B

Or, by the D line—

$$\left.\frac{40}{92^2}\right\{ 68 = ?$$

92 D : 40 C :: 68 D : 21.8 C

664. The top diameter of a vessel in the form of an elliptic cone is 53 inches by 44, the depth 24 inches: required the content.

$$53 \times 44 \times 24 \left\{ \begin{array}{l} \div 8472.87 \\ \\ \times .00011802 \end{array} \right\} = 6.6 \text{ bushels.}$$

By the slide-rule—

53 M D : 44 B :: 24 A : 25.3 B

And

42 A : 11 B :: 25.3 A : 6.6 B

665. A vessel in the form of a square prism is filled with malt, the depth is 28 inches, each side 36: required the content.

$$36^2 \times 28 \left\{ \begin{array}{l} \div 2118.192 \\ \\ \times .00045081 \end{array} \right\} = 16.35 \text{ bushels.}$$

By the slide-rule—

36 M D : 36 B :: 28 A : 16.35 B

Or, by the D line—

$$\frac{28 \times 36^2}{47.1^2} = ?$$

47.1 D : 28 C :: 36 D : 16.35 C

The gauge point, 47.1, will be found marked with a brass pin, and the letters M S, Malt, Square—that is, Bushels, Square; the content in bushels, the vessel square.

666. Required the content of a vessel in the form of a square pyramid, the sides of the top being each 63 inches, the depth 52.

$$63^2 \times 52 \left\{ \begin{array}{l} \div\ 6654.576 \\ \times\ .00015027 \end{array} \right\} = 31 \text{ bushels.}$$

By the slide-rule—

63 M D : 21 B :: 52 A : 31 B

The pyramid being $\frac{1}{3}$ of a prism, we take for the M D line $\frac{1}{3}$ of one of the dimensions, as if it were an oblong prism 63 by 21, and 52 deep.

Or thus, by the D line—

$$\left.\frac{52}{81.57^2}\right\} 63^2 = ?$$

81.57 D : 52 C :: 63 D : 31 C

667. It has been already stated that the duty is

charged upon the highest gauge; it is requisite, therefore, to reduce the various contents to net, to ascertain upon which the duty shall arise.

Examples.

668. The best cistern gauge of a steeping of barley was 96 bushels, of the couch 100 bushels, and of the floor 166.9 bushels: from which of the gauges does the charge arise, and what are the net bushels?

It is obvious the charge does not arise from the cistern, as it is less than the couch.

$$100 \times 1.63 = 163 \text{ couch.}$$
$$166.9 \text{ floor.}$$

Hence the duty is to be charged on the floor gauge, and

$$166.9 \times .5 = 83.45 \text{ bushels net.}$$

The best cistern and couch gauges are therefore cancelled in the Survey Book, and the best floor gauge is left.

669. The best couch gauge is 120.4 bushels, the best floor 195.4: from which of the gauges does the charge arise, and what are the net bushels?

$$120.4 \times .815 = 98.12 \text{ bushels net, by couch.}$$
$$195.4 \times .5 = 97.7 \quad \text{,,} \quad \text{floor.}$$

Hence the duty is charged on the couch gauge.

TIMBER MEASURING.

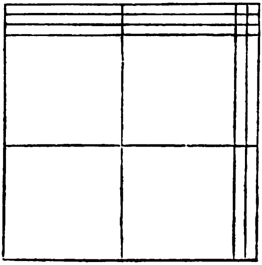
DUODECIMAL FRACTIONS.

670. Fractions are of three kinds—vulgar, decimal, and duodecimal. In vulgar fractions, unity may be divided into any number of equal parts; and the denominator must therefore be expressed as well as the numerator. In decimal fractions, unity is divided into successive tenths; and in duodecimal fractions, into successive twelfths; the denominator is therefore known, and, consequently, requires not to be written.

671. Duodecimal fractions are confined entirely to mensuration; and, as there are three kinds of measure—long, superficial, and solid—there are, necessarily, three distinct units—the long, or lineal unit; the superficial, or square unit; and the solid, or cubic unit; consequently, the successive duodecimals, or twelfths, all vary in their nature, according to the unit of which they are the fractions. The lineal unit is a lineal foot—a lineal unit in length; the superficial unit is a square foot—a lineal unit in length, and a lineal unit in breadth; and the solid unit is a cubic foot—a lineal unit in length, a lineal unit in breadth, and a lineal unit in thickness; consequently, the twelfth of a lineal unit is a lineal inch; the

twelfth of a superficial unit is a foot in length and an inch in breadth, and the twelfth of such twelfth is a square inch; the twelfth of a solid unit is a foot in length, a foot in breadth, and an inch in thickness; the twelfth of such twelfth is a foot in length, an inch in breadth, and an inch in thickness; and the twelfth of such twelfth of a twelfth is a cubic inch.

672. Suppose the following diagram to represent a rectangular surface $2\frac{3}{12}$ feet long, and $2\frac{2}{12}$ feet broad.



By vulgar fractions, we find the content thus—

$$2\tfrac{1}{4} \times 2\tfrac{1}{6} = \frac{\overset{3}{9}}{4} \times \frac{13}{\underset{2}{6}} = \frac{39}{8} = 4\tfrac{7}{8} \text{ square feet.}$$

By duodecimal fractions, thus—

$$\begin{array}{l}
2 + \frac{3}{12} \\
2 + \frac{2}{12} \\
\hline
4 + \frac{6}{12} \\
\quad + \frac{4}{12} + \frac{6}{144} \\
\hline
4 + \frac{10}{12} + \frac{6}{144} = 4\frac{126}{144} = 4\frac{7}{8} \text{ square feet.}
\end{array}$$

673. But, inasmuch as the denominators are known to be twelves, or powers of twelve,—as, in decimal fractions, they are tens, or powers of ten,—we omit them in practice, and write

Units or Feet.	Twelfths or Inches.	
2	3	
2	2	
4	6	
	4	6
4	10	6

4 units, or square feet; 10 twelfths, or foot-by-inch rectangles; 6 one-hundred-and-forty-fourths, or square inches.

674. This notation, antagonistic to the decimal, is attended with the difficulty of assigning an appropriate name (apart from the fractional) to the middle number, 10, which is neither feet nor inches, but a mixture of the two; 10 right-angled parallelograms, each a foot in length and an inch in breadth. Workmen term them inches, by which they understand inch breadths, the length being known to be the lineal unit. Some writers have proposed to call them

primes, but no appellation appears to have met with general adoption. The easiest way of explaining them is to attach the initials for feet and inches, and to write 4 F^2, 10 F I, 6 I^2; or to say 4 square feet and 126 square inches.

675. If we now suppose the above surface to be the top of a rectangular prism, the depth of which is 3 feet 7 inches; then, to find the content of it by vulgar fractions, we have

$$4\tfrac{7}{8} \times 3\tfrac{7}{12} = \frac{\overset{13}{\cancel{39}}}{8} \times \frac{43}{\underset{4}{\cancel{12}}} = \frac{559}{32} = 17\tfrac{15}{32} \text{ cubic feet.}$$

By duodecimal fractions—

$$\begin{array}{r}
4 + \frac{10}{12} + \frac{6}{144} \phantom{+ \frac{6}{1728}} \\
3 + \frac{7}{12} \phantom{+ \frac{6}{144} + \frac{6}{1728}} \\
\hline
14 + \frac{7}{12} + \frac{6}{144} \phantom{+ \frac{6}{1728}} \\
2 + \frac{10}{12} + \frac{1}{144} + \frac{6}{1728} \\
\hline
17 + \frac{5}{12} + \frac{7}{144} + \frac{6}{1728} \\
\hline
\end{array}$$

Or, without the denominators—

$$\begin{array}{rrrr}
4 & 10 & 6 & \\
3 & 7 & & \\
\hline
14 & 7 & 6 & \\
2 & 10 & 1 & 6 \\
\hline
17 & 5 & 7 & 6 \\
\hline
\end{array} = 17\tfrac{810}{1728} = 17\tfrac{15}{32} \text{ cubic feet.}$$

Where the difficulty of giving an appropriate name to the second and third numbers is greater still, the complication being increased by the greater variety of combinations. The easiest way to explain it is to write $17\ F^3$, $5\ F^2 I$, $7\ F I^2$, $6\ I^3$; or to say 17 cubic feet and 810 cubic inches. With the Slide-rule the content is given in cubic feet and tenths, by which all difficulty is avoided.

676. If it be desired to express the area of a surface in square yards, feet, and inches, it must be reduced subsequently, by dividing by the number of square feet contained in a square yard. Thus, suppose a rectangle is 7 feet 3 long by 6 feet 2 broad—

$$\begin{array}{rrr}
\text{Ft.} & \text{In.} & \\
7 & 3 & \\
6 & 2 & \\
\hline
43 & 6 & \\
1 & 2 & 6 \\
\hline
9|44 & 8 & 6 \\
\hline
\end{array}$$

4 sq. yds., 8 sq. ft., 102 sq. in.

677. We cannot, instead, multiply 2 yds. 1 ft. 3 in. by 2 yds. 2 in.; this would be no longer duodecimal fractions, but a mixture of thirds and twelfths, with their combinations, and would only lead to confusion, which, with solids, would be more complicated still. It is therefore to be understood that, in duodecimals, the unit is always a foot—lineal, superficial, or solid—of which the lower denominations are fractions, and the higher multiples.

678. For the mensuration of timber there are two rules in use—the timber rule, so-called, and the cubing rule. The former is for single pieces; the

latter chiefly for stacks or piles of timber. The cubing rule, being the simplest in construction, will be treated of first.

The Cubing Rule.

679. This rule is 3 feet in length, and contains four lines, marked A, B, C, and E. The A, C, and E lines are divided duodecimally—that is, into feet and inches; and the last is inverted, so that the numbers taken on the three lines are multiplied together; and, as the product of the three commencing numbers is placed against them on the commencement of the B line, the same is everywhere the case, and the rule gives the content at once in cubic feet. On one side of the rule the E line commences at 5 inches, and runs back to 12 feet. The 1, 2, 3, and 4 inches are supplied by the 10, 20, 30, and 40 of the B line, which numbers are marked with a star, to indicate that the 0 is to be rejected, and that they are to be taken as inches instead of feet. In reality, however, though they are marked on the B line, they are the numbers of the C line, for the 20 of the B line cuts across the 1 foot 8 inches of the C line, on which, as it works to the E line, the 2 inches ought properly to be taken. Now, 1 foot 8 inches = 20 inches; then, reckoning this 20 as 2, we reduce the product by taking $\frac{1}{10}$ of the number found, as will be exemplified

in the following examples. The second side of the rule is a continuation of the first, the one being for small numbers, the other for large. When none of the dimensions are under 5 inches, they are taken upon the lines in the order denoted by the letters E, C, A, B; when one is under 5 inches, and, consequently, marked with a star, by the letters E, B, A, B. The B line is divided decimally, giving the solidity in cubic feet and tenths, as before observed.

EXAMPLES.

680. A pile of battens is 12 feet long, 14 feet 3 inches high, and 9 feet 3 inches wide: required the content.

Ft.	In.			Ft.			Ft.	In.			Cub. ft.	
14	3	E	:	12	C	::	9	3	A	:	1581	B

681. A pile of deals is 14 feet long, 20 feet 7 inches wide, and 4 feet 7 inches high: required the content.

Ft.	In.			Ft.	In.			Ft.			Cub. ft.	
20	7	E	:	4	7	C	::	14	A	:	1320	B

682. A deal is 14 feet long, 2 feet 3 inches wide, and 3 inches thick: required the content.

Ft.	In.			In.			Ft.			Cub. Ft.	
2	3	E	:	3*	B	::	14	A	:	7.8	B

Here the 3* of B stands under the 30 inches of C, which 30 inches are, in reality, placed against the 2 feet 3 inches of E, and the product found on B is 78; but, as we reckon the 30 inches to be only 3, we take the content as $\frac{1}{10}$ of 78—viz., 7.8.

683. A plank is 14 feet long, 7 inches broad, and 2 inches thick: required the content.

In. In. Ft. Cub. ft.
7 E : 2* B :: 14 A : 1.3 B

684. A plank is 23 feet 3 inches long, 9 inches broad, and 4 inches thick: required the content.

In. In. Ft. In. Cub. ft.
9 E : 4* B :: 23 3 A : 5.8 B

685. A pile of staves is 4 feet 8 inches in length, 2 feet 7 inches in height, and 9 feet 9 inches in width: required the content.

Ft. In. Ft. In. Ft. In. Cub. ft.
2 7 E : 4 8 C :: 9 9 A : 117 B

686. A frame or pile of lathwood is 8 feet 1 inch in length, 6 feet in height, and 6 feet in width: required the content.

Ft. Ft. Ft. In. Cub. ft.
6 E : 6 C :: 8 1 A : 291 B

687. A pile of balks is 24 feet 3 inches in length, 3 feet 6 inches in height, and 3 feet in width: required the content.

Ft.		Ft.	In.		Ft.	In.		Cub. ft.
3 E	:	3	6 C	::	24	3 A	:	254 B

When the divisions, as in this case, fall near the end of the line, where they are rather crowded, the content is computed with greater exactness by taking the length in two operations, a portion at a time; thus—

Ft.		Ft.	In.		Ft.	In.		Cub. ft.
3 E	:	3	6 C	::	20	0 A	:	210 B
				::	4	3 A	:	44.6 B
								254.6

688. The same course, it is obvious, is to be adopted if the dimensions are too large to be taken on the rule at once.

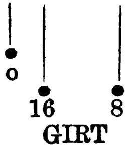
EXAMPLE.

689. A pile of timber is 42 feet 6 inches in length, 5 feet 9 inches in height, and 6 feet 10 inches in width: required the content.

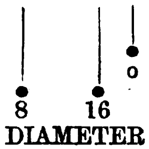
Ft.	In.		Ft.	In.		Ft.	In.		Cub. ft.
30	0 E	:	5	9 C	::	6	10 A	:	1178 B
12	6 E	:			::			:	491 B
									1669

THE TIMBER RULE.

690. This is usually two feet long. One side contains four lines, A, B, C, and E, the last of which is inverted, and so graduated that 144 falls opposite the commencement of the slide; consequently, it divides the product by 144, and reduces square inches to square feet. Therefore, by taking the length of a piece of timber in feet, and the thickness and width in inches, it gives the content in cubic feet. The dimensions are taken in the order of the letters E, C, A, B; the last, or B line, exhibiting the content. The A, C, and E lines are divided into fourths, the B line decimally. The other side of the rule contains the usual A, B, C, and D lines, the last of which, termed the girt line, is made to commence at 4 and run on to 40, as on the common Carpenter's rule. On this face of the rule are eight gauge points, marked with brass pins; one on the A line at 12 for reducing lineal inches to lineal feet; one on the D line at 12 for reducing square inches to square feet; three between the 10 and 11, marked



And three between the 13 and 14, marked



691. The o indicates the gauge points for round timber; the 16 for hexdecagonal, or sixteen-sided timber; and the 8 for octagonal, or eight-sided timber.

692. The words Girt and Diameter explain themselves; the former denoting the measure round, the latter across. As applied to octagonal and hexdecagonal timber, by diameter is to be understood the perpendicular distance between opposite sides, in contradistinction to diagonal, which is the distance between opposite angles; or, as applied to polygons in general, with an even or an odd number of sides, the diameter is equal to the diameter of the inscribed circle; the diagonal, to the diameter of the circumscribing circle. Instead of GIRT, however, it ought to be printed QR. GIRT; for, while with the diameter gauge points *the whole* of the diameter is taken, with the girt gauge points *only one-fourth* of the GIRT is taken, to suit the numbers on the rule; so that, in reality, they are Quarter-girt gauge points, and should be so marked.

693. The dimensions being taken in inches, the gauge points for reducing the content to square feet are determined as follows:—

694. The side of an octagon being 1, the side of the circumscribing square, or the diameter of the polygon, is $= \sqrt{2} + 1$; hence

$$\begin{array}{lr}
\text{Area of square} = \overline{\sqrt{2} + 1}|^2 & = 2\sqrt{2} + 3 \\
\text{Area of 4 corner triangles cut off} = & 1 \\
\hline
\text{Area of octagon} & = 2\sqrt{2} + 2 \\
\hline
\end{array}$$

695. The side of a hexdecagon being 1, the side of the circumscribing square, or the diameter of the polygon, is = tan 78° 45′; and the area = 4 tan 78° 45′.

696. The diameter of a circle being 1, the area is = .7854.

697. Collecting these into a tabular form, we have

Polygon.	Perimeter.	Qr. Girt.	Diameter.	Area.
Octagon....	8.	2.	$\sqrt{2} + 1$	$2\sqrt{2} + 2$
Hexdecagon.	16.	4.	tan 78° 45′	4 tan 78° 45′
Circle......	3.1416	.7854	1.	.7854

698. Then, since the areas are as the squares of

the diameters or perimeters, to find the gauge points, we have

For the octagon—

$$2\sqrt{2} + 2 : 2\sqrt{2} + 3 :: 144$$

Or, $$\sqrt{2} + 1 : 2\sqrt{2} + 3 :: 72$$

Or (multiplying 1st and 2nd terms by $\sqrt{2} - 1$),

$1 : \sqrt{2} + 1 :: 72 : 72(\sqrt{2} + 1) = 173.8233$, the square root of which is 13.18, the octagonal diameter gauge point.

Also,

$$2\sqrt{2} + 2 : 2^2 :: 144$$

Or, $$\sqrt{2} + 1 : 2 :: 144$$

Or (multiplying 1st and 2nd terms by $\sqrt{2} - 1$),

$1 : 2(\sqrt{2} - 1) :: 144 : 288(\sqrt{2} - 1) = 119.2935$, the square root of which is 10.92, the octagonal quarter-girt gauge point.

For the hexdecagon—

$$\tan 78^\circ 45' \times 4 : \tan^2 78^\circ 45'$$

Or, $$4 : \tan 78^\circ 45'$$

Viz., $$4 : 5.02734 :: 144 : 180.9842,$$

the square root of which is 13.45, the hexdecagonal diameter gauge point.

Also, 4 tan 78° 45′ : 4^2

Or, tan 78° 45′ : 4

Viz., 5.02734 : 4 :: 144 : 114.5735,

the square root of which is 10.7, the hexdecagonal quarter-girt gauge point.

For the circle—

.7854 : 1^2 :: 144 : 183.3464,

the square root of which is 13.54, the circular diameter gauge point.

Also, .7854 : $.7854^2$

Or, 1 : .7854 :: 144 : 113.0976,

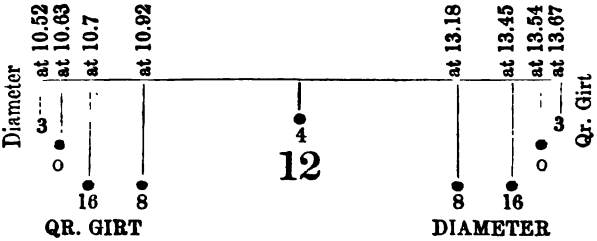
the square root of which is 10.63, the circular quarter-girt gauge point.

699. Arranging them into a tabular form, they will be as follows:—

Table of Divisors, Factors, and Gauge Points, for Areas.

Dimensions in Inches. Content in Square Feet.	POLYGONS.		
	Octagon.	Hexdecagon.	Circle.
DIAMETER.			
Divisors	173.8233	180.9842	183.3464
Factors	.0057529	.0055253	.00545415
Gauge Points ..	13.18^2	13.45^2	13.54^2
QR. GIRT.			
Divisors	119.2935	114.5735	113.0976
Factors	.0083826	.00872804	.00884192
Gauge Points ..	10.92^2	10.7^2	10.63^2

Hence the gauge points of the D line stand thus—



700. There is a beautiful order in the arrangement and position of these gauge points. The circle, on either hand, is the limit of all polygons, the gauge points of which approach towards each other, as the sides diminish in number, till they meet in the square, when the quarter-girt and diameter become equal. The only remaining polygon is the equilateral triangle, the quarter-girt of which exceeds the diameter of the inscribed circle; the gauge points, accordingly, cross over, and the diameter gauge point falls to the left of the quarter-girt at 10.52; while the quarter-girt gauge point runs to the right of the diameter at 13.67; and so they go on, either way, 3, 4, 8, 16, to infinity, the circle. The gauge points for the equilateral triangle are not graduated on the rule, but are exhibited above to include the entire range of polygons; nor is the 4 placed beneath the square gauge point, but is shown here, to render the explanation clearer.

In section 111 are given the areas of polygons whose sides are unity; they are, otherwise, thus represented:—

701. Putting n = the number of sides in the polygon, the area $= \frac{n}{4} \tan \frac{(n-2)90^\circ}{n}$; which formula arranges itself for the various polygons as follows:—

Polygon.	Area.
Trigon	$\frac{3}{4} \tan 30^\circ = \frac{3}{4} \times .577350$
Tetragon......	$\frac{4}{4} \tan 45^\circ = \frac{4}{4} \times 1.000000$
Pentagon ...	$\frac{5}{4} \tan 54^\circ = \frac{5}{4} \times 1.376382$
Hexagon......	$\frac{6}{4} \tan 60^\circ = \frac{6}{4} \times 1.732051$
Heptagon ...	$\frac{7}{4} \tan 64\frac{2}{7}^\circ = \frac{7}{4} \times 2.076521$
Octagon	$\frac{8}{4} \tan 67\frac{1}{2}^\circ = \frac{8}{4} \times 2.414214$
Nonagon......	$\frac{9}{4} \tan 70^\circ = \frac{9}{4} \times 2.747477$
Decagon	$\frac{10}{4} \tan 72^\circ = \frac{10}{4} \times 3.077684$
Undecagon ...	$\frac{11}{4} \tan 73\frac{7}{11}^\circ = \frac{11}{4} \times 3.405688$
Dodecagon ...	$\frac{12}{4} \tan 75^\circ = \frac{12}{4} \times 3.732051$
Hexdecagon	$\frac{16}{4} \tan 78\frac{3}{4}^\circ = \frac{16}{4} \times 5.027340$

Examples with the Timber Rule.

702. To divide by 144, use the inverted E line, which commences at that number.

To divide by 12, use the $\underset{|}{\overset{12}{\bullet}}$ of the A line.

To divide by 12^2, use the $\underset{12}{\overset{|}{\bullet}}$ of the D line.

703. The length of a plank is $25\frac{1}{4}$ feet, the mean breadth, or breadth in the middle, $16\frac{1}{4}$ inches; the mean thickness $4\frac{1}{2}$ inches: required the content in cubic feet.

$$\frac{25\frac{1}{4} \times 16\frac{1}{4} \times 4\frac{1}{2}}{144}$$

$$4\tfrac{1}{2}\,\mathrm{E} : 16\tfrac{1}{4}\,\mathrm{C} :: 25\tfrac{1}{4}\,\mathrm{A} : 12.8\,\mathrm{B}$$

704. A plank is 7 feet long, 19 inches broad: how many superficial feet does it contain?

$$\frac{7 \times 19}{12}$$

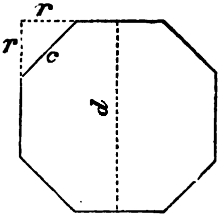
$$\underset{|}{\overset{12}{\bullet}}\,\mathrm{A} : 7\,\mathrm{B} :: 19\,\mathrm{A} : 11\,\mathrm{B}$$

705. A board is 20 inches square: how many superficial feet does it contain?

$$\frac{1 \times 20^2}{12^2}$$

$$\underset{12}{\overset{|}{\bullet}}\,\mathrm{D} : 1\,\mathrm{C} :: 20\,\mathrm{D} : 2.77\,\mathrm{C}$$

706. To find the area of a square having 4 equal triangular corners cut off, as annexed.



$$c^2 = 2\,r^2$$

But $2\,r^2$ = the content of the 4 triangles.

$\therefore\ c^2$ = the content of the 4 triangles.

Hence $d^2 - c^2$ = the content of the figure.

707. The diameter, *d*, of a square is 18½ inches; the chord of the defective angle, *c*, is 5 inches: required the area in square feet.

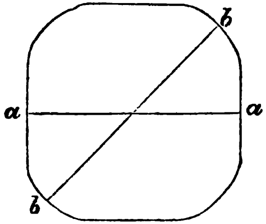
$$\frac{1 \times (18.5^2 - 5^2)}{12^2} = ?$$

$\dot{12}$ D : 1 C :: 18.5 D : 2.37 $\dot{\text{C}}$
5. D : .17 C

2.2

708. To find the area of a square having the 4 corners rounded off.

Take the diameter by the square side *a a*, and by the circular side *b b*; add the two together, and take half the sum for a mean diameter; then use the circular diameter gauge point, considering the figure as round.



709. A square, with the corners rounded off, measures 18 inches diameter by the square side, 19 by the circular side: required the content in square feet.

$$\frac{18 + 19}{2} = 18.5 \text{ mean diameter.}$$

$$\text{Then } \frac{1 \times 18.5^2}{13.54^2} = ?$$

$$\underset{\circ}{!}\ \text{D} : 1\ \text{C} :: 18\tfrac{1}{2}\ \text{D} : 1.86\ \text{C}$$

710. Octagonal masts by the diameter.

$$\text{Formula, } \frac{\text{L}\,d^2}{173.8233}\text{; or, } \frac{\text{L}\,d^2}{13.18^2}$$

An octagonal mast is 70½ feet long, 20½ inches diameter: required the content in cubic feet.

$$\underset{8}{\text{!}}\ \text{D} : 70\tfrac{1}{2}\ \text{C} :: 20\tfrac{1}{2}\ \text{D} : 170\ \text{C}$$

Examples for Practice.

Length.	Diameter.	Content.
73 ft.	20¾ in.	181 cubic ft.
72	21¼	187
88	23½	280

711. Octagonal masts by the quarter-girt.

$$\text{Formula}\quad \frac{\text{L}\,.\,\overline{\text{qr. g.}}|^{2}}{119.2935};\ \text{or,}\ \frac{\text{L}\,.\,\overline{\text{qr. g.}}|^{2}}{10.92^{2}}$$

An octagonal mast is 72½ feet long, the quarter-girt 18½ inches: required the content in cubic feet.

$$\underset{8}{\text{!}}\ \text{D} : 72\tfrac{1}{2}\ \text{C} :: 18\tfrac{1}{2}\ \text{D} : 208\ \text{C}$$

Examples for Practice.

Length.	Qr. Girt.	Content.
75½ ft.	19¼ in.	234 cubic ft.
73	20½	257
72	20¾	260

712. Hexdecagonal masts by the diameter.

$$\text{Formula}\quad \frac{\text{L}\,d^{2}}{180.9842};\ \text{or,}\ \frac{\text{L}\,d^{2}}{13.45^{2}}$$

A hexdecagonal mast is 82½ feet long, 21½ inches diameter: required the content in cubic feet.

$$\underset{16}{\overset{|}{\bullet}}\ \mathrm{D} : 82\tfrac{1}{2}\ \mathrm{C} :: 21\tfrac{1}{2}\ \mathrm{D} : 210\tfrac{1}{2}\ \mathrm{C}$$

EXAMPLES FOR PRACTICE.

Length.	Diameter.	Content.
81 ft.	24 in.	258 cubic ft.
79	20¼	179
89½	25½	322

713. Hexdecagonal masts by the quarter-girt.

$$\text{Formula}\quad \frac{\mathrm{L}\,.\,\overline{\text{qr. g.}}\rceil^2}{114.5735}\,;\ \text{or,}\ \frac{\mathrm{L}\,.\,\overline{\text{qr. g.}}\rceil^2}{10.7^2}$$

A hexdecagonal mast is 74 feet long, the quarter-girt is 17¼ inches: required the content in cubic feet.

$$\underset{16}{\overset{|}{\bullet}}\ \mathrm{D} : 74\ \mathrm{C} :: 17\tfrac{1}{4}\ \mathrm{D} : 192\ \mathrm{C}$$

EXAMPLES FOR PRACTICE.

Length.	Qr. Girt.	Content.
72½ ft.	17¼ in.	188 cubic ft.
73	17½	195
77	18¾	236

714. Round masts by the diameter.

$$\text{Formula}\quad \frac{\mathrm{L}\ d^2}{183.3464}\,;\ \text{or,}\ \frac{\mathrm{L}\ d^2}{13.54^2}$$

A round mast is 80½ feet long, 20½ inches diameter: required the content in cubic feet.

$$\underset{\text{o}}{\overset{|}{\bullet}}\ \text{D} : 80\tfrac{1}{2}\ \text{C} :: 20\tfrac{1}{2}\ \text{D} : 184\ \text{C}$$

Examples for Practice.

Length.	Diameter.	Content.
81 ft.	21¼ in.	199 cubic ft.
82	22½	226
85	23¾	261

715. **Round masts by the quarter-girt.**

$$\text{Formula}\quad \frac{\text{L}.\overline{\text{qr. g.}})^2}{113.0976};\ \text{or,}\ \frac{\text{L}.\overline{\text{qr. g.}})^2}{10.63^2}$$

A round mast is 80½ feet long, the quarter-girt is 19 inches: required the content.

$$\underset{\text{o}}{\overset{|}{\bullet}}\ \text{D} : 80\tfrac{1}{2}\ \text{C} :: 19\ \text{D} : 257\ \text{C}$$

Examples for Practice.

Length.	Qr. Girt.	Content.
78 ft.	18½ in.	236 cubic ft.
79	19¼	259
82	18¾	255

716. **Irregular square masts with defective angles.**

$$\text{Formula}\quad \frac{\text{L}(d^2 - c^2)}{144};\ \text{or,}\ \frac{\text{L}(d^2 - c^2)}{12^2}$$

An irregular square mast is 70 feet long; the diameter, *d*, is 18½ inches; the chord, *c*, of the defective angle is 5 inches: required the content in cubic feet.

$$\overset{\mid}{\dot{1}2}\text{ D : 70 C :: } 18\tfrac{1}{2}\text{ D : 166 C}$$
$$\text{:: 5 D : 12 C}$$
$$\underline{}$$
$$\text{154 cubic ft.}$$

EXAMPLES FOR PRACTICE.

Length.	Diameter.	Chord of Defective Angle.	Content.
72 ft.	19¼ in.	4 in.	177 cubic ft.
73	18¾	4½	168
75	19	6¾	164
76	19¼	5½	179

717. The diameter of masts is to be taken with the callipers at the middle of the mast. The chord of the defective angle is to be measured at the same place, as is also the circumference or perimeter, by passing a measuring-tape round it, one-fourth of which circumference or perimeter is the quarter-girt.

718. A bowsprit is made up of the following figures:—first, an irregular square heel, 23½ feet long, the diameter of the square 24 inches, and the chord of the defective angle 3 inches; secondly, an

octagonal middle, 30 feet in length and 25 inches in diameter; and, thirdly, a square head, 10 feet in length, and the side of the square, 18½ inches: required the content in cubic feet.

Irregular square heel	=	92½
Octagonal middle	=	108
Square head	=	23¾
		224¼ cubic ft.

719. A topmast is made up of the four following figures:—First, a square heel, 9 feet in length and 18 inches square; secondly, an octagonal part, 16 feet in length and 16½ inches in diameter; thirdly, a round part, 14 feet in length and 15 inches in diameter; and, lastly, a square head, 6 feet in length and 12 inches square: required the content in cubic feet.

Square heel	=	20
Octagon	=	25
Round	=	17
Square head	=	6
		68 cubic ft.

ON THE USE OF THE TABLES.

720. By preparing a table for the areas of squares, and another for those of circles, we are enabled to ascertain the capacities of solids in a much readier way than by the usual process of calculation. The labour might be still farther abridged by computing a separate table for each of the subratios at page 78. Some of the solids, however, to which they relate occur so seldom, that to insert them would only swell the work, without any commensurate advantage. The most convenient, upon the whole, are, for the solids of ascension, a table of the areas of semi-squares; and, for the solids of rotation, the areas of circles: to which may be added, for the 1st and 2nd varieties of casks, tables of two-third areas of circles, of one-third areas of circles, and of two-fifteenth areas of circles. The mode of using them will be understood by the following examples:—

Table I.

721. This contains, in gallons, the areas of semi-squares, or half-areas of squares, the dimensions being

taken inches. It is formed by squaring the side, dividing the product by 277.274, and taking one-half for the tabular number or semi-square.

For Squares.

722. **Rule.**—Take out the number standing against the side and double it; the product is the area.

Examples.

723. Required the area of the square in Question 213.

$$\begin{array}{rrl} 60 = & 6.4918 & = \frac{1}{2} \text{ area.} \\ & 2 & \\ \hline & 12.9836 & \text{gallons.} \\ \hline \end{array}$$

724. Capacity of square prism, Question 315.

$$\begin{array}{rrl} 30 = & 1.6229 & = \frac{1}{2} \text{ area.} \\ & 100 & = \text{twice depth.} \\ \hline & 162.29 & \text{gallons.} \\ \hline \end{array}$$

725. Square pyramid, Question 342.

The subratio of the pyramid is $\frac{1}{3}$; therefore, multiply the area by $\frac{1}{3}$ the height, or the half-area by $\frac{2}{3}$ the height.

$$\begin{array}{rrl} 25 = & 1.1270 & = \frac{1}{2} \text{ area.} \\ & 62 & = \frac{2}{3} \text{ height.} \\ \hline & 22540 & \\ & 67620 & \\ \hline & 69.8740 & \text{gallons.} \\ \hline \end{array}$$

726. **Frustum of square pyramid, Question 383.**

$$\begin{array}{llrl} \text{Top} & . & 22 = & .8728 \\ \text{Bottom} & & 16 = & .4616 \\ \text{Sum} & . & 38 = & 2.6039 \\ \hline & & & 3.9383 = 6 \text{ half-areas} = 3 \text{ areas.} \\ & & & \quad 20 \\ & & & 3\overline{)78.7660} = 3 \text{ capacities.} \\ \hline & & & 26.2553 \end{array}$$

For Parallelograms.

727. The area of a parallelogram is the product of the length and breadth.

Now, if from the sum of the semi-squares of any two numbers we subtract the semi-square of their difference, the remainder is the product of those two numbers. For

$$\begin{array}{rcl} \frac{1}{2}(m^2 + n^2) &=& \frac{1}{2} m^2 + \frac{1}{2} n^2 \\ \frac{1}{2}(m - n)^2 &=& \frac{1}{2} m^2 + \frac{1}{2} n^2 - m n \\ \hline \text{difference} &=& \qquad\qquad + m n \end{array}$$

Hence the area is obtained by the following

728. **Rule.**—Add together the tabular semi-squares of the length and breadth, and from the sum subtract the tabular semi-square of their difference. The remainder is the area of the parallelogram required.

EXAMPLES.

729. Area of parallelogram, Question 218.

$$\begin{array}{rcr} 50 & = & 4.5082 \\ 30 & = & 1.6229 \\ \hline & & 6.1311 \\ 20 & = & .7213 \\ \hline & & \underline{5.4098} \text{ gallons.} \end{array}$$

730. Area of rhombus, Question 219.

$$\begin{array}{rcr} 40 & = & 2.8852 \\ 37 & = & 2.4687 \\ \hline & & 5.3539 \\ 3 & = & .0162 \\ \hline & & \underline{5.3377} \text{ gallons.} \end{array}$$

731. Area of triangle, Question 223.

The subratio of the triangle is $\frac{1}{2}$; therefore, take $\frac{1}{2}$ one of the dimensions.

$$\begin{array}{rrcr} \frac{1}{2} \text{ base} = & 30 & = & 1.6229 \\ & 23.5 & = & .9959 \\ \hline & & & 2.6188 \\ & 6.5 & = & .0762 \\ \hline & & & \underline{2.5426} \text{ gallons.} \end{array}$$

732. Area of parabola, Question 278.

The subratio of the parabola is $\frac{2}{3}$; therefore, take $\frac{2}{3}$ one of the dimensions.

$$\begin{array}{rr}
 & 45 = 3.6516 \\
\frac{2}{3}\text{ base} = & 40 = 2.8852 \\
\hline
 & 6.5368 \\
 & 5 = .0451 \\
\hline
 & 6.4917 \text{ gallons.}
\end{array}$$

733. Area of sector, Question 244.

The sector may be considered as a triangle with a circular base; consequently, the same rule applies to both; therefore, take $\frac{1}{2}$ one of the dimensions.

$$\begin{array}{rr}
 & 32 = 1.8465 \\
\frac{1}{2}\text{ radius} = & 20 = .7213 \\
\hline
 & 2.5678 \\
 & 12 = .2597 \\
\hline
 & 2.3081 \text{ gallons.}
\end{array}$$

734. Capacity of oblong pyramid, Question 344.

The subratio of the pyramid is $\frac{1}{3}$; therefore, take $\frac{1}{2}$ one of the dimensions.

$$\begin{array}{r}
36 = 2.3370 \\
15 = .4057 \\
\hline
2.7427 \\
21 = .7952 \\
\hline
1.9475 \\
50 \\
\hline
97.3750 \text{ gallons.}
\end{array}$$

735. Prismoid, Question 658.

$$\begin{array}{lrrl}
\text{Top} & = 70 \times 60 & \left\{\begin{array}{r}8.8360\\6.4918\end{array}\right. & \\
\text{Bottom} & = 100 \times 80 & \left\{\begin{array}{r}18.0327\\11.5409\end{array}\right. & \\
\text{Sum} & = 170 \times 140 & \left\{\begin{array}{r}52.1145\\35.3441\end{array}\right. & \\
\cline{3-3}
 & & 132.3600 & \\
 & & 2.5245 & \\
\cline{3-3}
 & & 129.8355 & = 6 \text{ areas.}\\
 & & 1.5 & = \frac{1}{6} \text{ depth.}\\
\cline{3-3}
 & & 6491775 & \\
 & & 1298355 & \\
\cline{3-3}
 & & 8\,|\,194.75325 & \text{gallons.}\\
\cline{3-3}
 & & 24.34 & \text{bushels.}
\end{array}$$

$$\begin{array}{rcr}
\multicolumn{3}{r}{\text{Differences}}\\
10 & = & .1803\\
20 & = & .7213\\
30 & = & 1.6229\\
\cline{3-3}
 & & 2.5245
\end{array}$$

736. Wedge, Question 660.

$$\begin{array}{lrrl}
\text{Top} & = 48 \times 0 & & \\
\text{Bottom} & = 60 \times 11 & \left\{\begin{array}{r}6.4918\\.2182\end{array}\right. & \\
\text{Sum} & = 108 \times 11 & \left\{\begin{array}{r}21.0333\\.2182\end{array}\right. & \\
\cline{3-3}
 & & 27.9615 & \\
 & & 21.2967 & \\
\cline{3-3}
 & & 6\,|\,6.6648 & = 6 \text{ areas.}\\
\cline{3-3}
 & & 1.1108 & \\
 & & 20 & \\
\cline{3-3}
 & & 8\,|\,22.2160 & \text{gallons.}\\
\cline{3-3}
 & & 2.777 & \text{bushels.}
\end{array}$$

$$\begin{array}{rcr}
\multicolumn{3}{r}{\text{Differences.}}\\
48 & = & \\
49 & = & 4.3297\\
97 & = & 16.9670\\
\cline{3-3}
 & & 21.2967
\end{array}$$

As the 48 occurs on each side, one cancels the other; so the tabular areas for both are omitted.

TABLE II.

737. This contains, in gallons, the areas of circles, the dimensions being taken in inches. It is formed by squaring the diameter, and dividing the product by 353.036.

FOR CIRCLES.

738. RULE.—Look in the table for the diameter. The number standing against it is the area.

EXAMPLES.

739. Required the area of circle in Question 238.

$$45 = 5.7359 \text{ gallons.}$$

740. Capacity of cylinder, Question 333.

$$\begin{array}{r} 21.5 = 1.3093 \\ 40 \\ \hline \underline{52.3720} \text{ gallons.} \end{array}$$

741. Capacity of sphere, Question 361.

The subratio of the sphere or spheroid is $\frac{2}{3}$; therefore, take $\frac{2}{3}$ the fixed axe.

$$\begin{array}{r} 20 = 1.1330 \\ 13\frac{1}{3} = \frac{2}{3} \text{ diameter.} \\ \hline 147290 \\ 3776 \\ \hline \underline{15.1066} \text{ gallons.} \end{array}$$

742. Prolate spheroid, Question 363.

$$\begin{array}{rl} 33 = 3.0847 & \\ 36 & = \frac{2}{3} \text{ fixed axe.} \\ \hline 185082 & \\ 92541 & \\ \hline 111.0492 & \text{gallons.} \\ \hline \end{array}$$

743. Oblate spheroid, Question 364.

$$\begin{array}{rl} 54 = 8.2597 & \\ 22 & = \frac{2}{3} \text{ fixed axe.} \\ \hline 165194 & \\ 165194 & \\ \hline 181.7134 & \text{gallons.} \\ \hline \end{array}$$

744. Cone, Question 352.

The subratio of the cone is $\frac{1}{3}$; therefore, take $\frac{1}{3}$ the fixed axe.

$$\begin{array}{rl} 46 = 5.9937 & \\ 13 & = \frac{1}{3} \text{ depth.} \\ \hline 77.9181 & \text{gallons.} \\ \hline \end{array}$$

745. Paraboloid, Question 366.

The subratio of the paraboloid is $\frac{1}{2}$; therefore, take $\frac{1}{2}$ the fixed axe.

$$\begin{array}{rl} 82 = 19.0462 & \\ 48 & = \frac{1}{2} \text{ depth.} \\ \hline 1523696 & \\ 761848 & \\ \hline 914.2176 & \text{gallons.} \\ \hline \end{array}$$

746. Hyperboloid, Question 373.

The divisor given in the rule at page 133 is that of the round frustum, the subratio of which is $\frac{1}{6}$; therefore, take $\frac{1}{6}$ the fixed axe.

52 = 7.6592
68 = 13.0978
20.7570 = 6 areas.
50
6|1037.8500 = 6 capacities.
172.975 gallons.

747. Spindle, Question 378.

28 = 2.2207
40 = 4.5321
6.7528 = 6 areas.
70
6|472.6960 = 6 capacities.
78.782 gallons.

748. Frustum of cone, Question 389.

Top . 20 = 1.1330
Bottom 36 = 3.6710
Sum . 56 = 8.8829
13.6869 = 6 areas.
43
410607
547476
6|588.5367 = 6 capacities.
98.0894 gallons.

749. Parabolic spindle, Question 369.

The subratio of the parabolic spindle is $\frac{8}{15}$; therefore, take $\frac{8}{15}$ of the fixed axe; or multiply by the fixed axe, and take $\frac{8}{15}$ of the product.

28 = 2.2207
70
155.4490 circumscribing cylinder.
8
15|1243.5920
82.906 gallons.

750. Frustum of paraboloid, Question 395.

30 = 2.5493
58 = 9.5287
12.0780 = 2 areas.
9
2|108.7020 = 2 capacities.
54.351 gallons.

751. Frustum of hyperboloid, Question 398.

23 = 1.4984
40 = 4.5321
72 = 14.6840
20.7145 = 6 areas.
20
6|414.2900 = 6 capacities.
69.048 gallons.

752. Middle frustum of spindle, Question 410.

$$\begin{array}{rrl}
24 = & 1.6315 & \\
32 = & 2.9005 & \\
60 = & 10.1972 & \\
\hline
 & 14.7292 & = 6 \text{ areas.} \\
 & 40 & \\
\hline
6 \,| & 589.1680 & = 6 \text{ capacities.} \\
\hline
 & \underline{98.194} & \text{gallons.}
\end{array}$$

753. Middle zone of sphere, Question 421.

$$\begin{array}{rr l}
22 = & & 1.3709 \\
10 = .2832, \div 3 = & & .0944 \\
\hline
 & & 1.2765 \\
 & & 10 \\
\hline
 & & \underline{12.765} \text{ gallons.}
\end{array}$$

754. Frustum of sphere, Question 426.

The divisor given in the rule at page 163 is for the paraboloid, the subratio of which is $\frac{1}{2}$; therefore, take $\frac{1}{2}$ the fixed axe.

$$\begin{array}{rrl}
36 = & 3.6710 & \\
42 = & 4.9966 & \\
30 = & 2.5493 & \\
+\frac{1}{3} = & .8497\frac{2}{3} & \\
\hline
 & 12.0666\frac{2}{3} & = 2 \text{ areas.} \\
 & 15 & \\
\hline
 & \underline{181.0000} & \text{gallons.}
\end{array}$$

755. Circular vessel, Question 452.

4	2	1
83 = 19.5135	86 = 20.9496	80 = 18.1284
88 = 21.9354	90 = 22.9438	81 = 18.5844
89 = 22.4367	87 = 21.4397	36.7128
84 = 19.9866	65.3331	
83.8722	130.6662	
335.4888		
130.6662		
36.7128		
502.8678		

4 = $\frac{1}{3}$ common distance.

2011.4712 gallons.

For Ellipses.

756. The ellipse bears the same relation to the circle as the parallelogram to the square; but, as Table 1 is computed for *half square* areas, while Table 2 is for *whole circular* areas, the rule for ellipses is somewhat different from that for parallelograms.

If from the square of half the sum of any two numbers we subtract the square of half their difference, the remainder is the product of those two numbers. For

$$\begin{array}{rcl}(\frac{1}{2}m + \frac{1}{2}n)^2 & = & \frac{1}{4}m^2 + \frac{1}{4}n^2 + \frac{1}{2}mn \\ (\frac{1}{2}m - \frac{1}{2}n)^2 & = & \frac{1}{4}m^2 + \frac{1}{4}n^2 - \frac{1}{2}mn \\ \hline \text{difference} & = & \qquad\qquad\qquad + \quad mn\end{array}$$

Hence the area of ellipses is obtained by the following

757. **Rule.**—From the tabular area corresponding to half the sum of the two diameters, deduct the tabular area answering to half their difference; the remainder is the area of the ellipse.

Examples.

758. Required the area of the ellipse in Question 264.

$$\frac{24 \pm 18}{2} = \left\{ \begin{array}{rl} 21 = & 1.2491 \\ 3 = & .0254 \\ \hline & 1.2237 \text{ gallons.} \end{array} \right.$$

759. Elliptic cylinder, Question 338.

$$\frac{40 \pm 32}{2} = \left\{ \begin{array}{rr} 36 = & 3.6710 \\ 4 = & .0453 \\ \hline & 3.6257 \\ & 15 \\ \hline & 54.3855 \text{ gallons.} \end{array} \right.$$

760. Elliptic cone, Question 355.

$$\frac{12 \pm 10}{2} = \left\{ \begin{array}{rr} 11 = & .3427 \\ 1 = & .0028 \\ \hline & .3399 \\ & 8 = \frac{1}{3} \text{ depth.} \\ \hline & 2.7192 \text{ gallons.} \end{array} \right.$$

TABLE III.

761. The areas in this table are $\frac{2}{3}$ of those in Table 2.

TABLE IV.

762. The areas in this table are $\frac{1}{3}$ of those in Table 2.

TABLE V.

763. The areas in this table are $\frac{2}{15}$ of those in Table 2.

They are constructed for finding the capacities of casks. The mode of using them is as follows:—

EXAMPLES.

764. Required the content of the cask in Question 492, for the 4 varieties.

Substituting divisors for gauge points, and putting $c = 353.036$, the formulæ become

1st. Pro. sphd. $\dfrac{L(2\,B^2 + H^2)}{3\,c}$

2nd. Par. spin. $\dfrac{L(2\,B^2 + H^2 - \frac{2}{5}\,\text{Diff.}^2)}{3\,c}$

3rd. Two parab. $\dfrac{L(B^2 + H^2)}{2\,c}$

4th. Two co. $\dfrac{L(B^2 + H^2 + \overline{B + H}|^2)}{6\,c}$

765. Omitting the *c*, as being the divisor by which the tables are computed, they may be arranged as follows:—

1st. Pro. sphd. $L(\frac{2}{3}B^2 + \frac{1}{3}H^2)$

2nd. Par. spin. $L(\frac{2}{3}B^2 + \frac{1}{3}H^2 - \frac{2}{15}\text{Diff.}^2)$

3rd. Two parab. $\frac{L}{2}(B^2 + H^2)$

4th. Two co. $\frac{L}{6}(B^2 + H^2 + \text{Sum}^2)$

766. Hence, for the 1st and 2nd varieties, we select the tabular numbers for the bung from the $\frac{2}{3}$ areas of Table 3, for the head from the $\frac{1}{3}$ areas of Table 4, for the Diff. from the $\frac{2}{15}$ areas of Table 5. Had we tables of $\frac{1}{2}$ areas and $\frac{1}{6}$ areas, we might use them for the 3rd and 4th varieties. As we have not, we take $\frac{1}{2}$ the length and $\frac{1}{6}$ the length instead.

767. *First variety.*

$$\begin{array}{lr} \text{Table 3, B} = 32 = & 1.9337 \\ \text{Table 4, H} = 24 = & .5439 \\ \hline & 2.4776 \\ & 40 \\ \hline & 99.1040 \text{ gallons.} \end{array}$$

768. *Second variety.*

$$\begin{array}{llr}
\text{Table 3,} & B = 32 = & 1.9337 \\
\text{Table 4,} & H = 24 = & .5439 \\
\hline
 & & 2.4776 \\
\text{Table 5,} & \text{Diff. } 8 = & .02417 \\
\hline
 & & 2.45343 \\
 & & 40 \\
\hline
 & & 98.13720 \text{ gallons.} \\
\hline
\end{array}$$

769. *Third variety.*

$$\begin{array}{llr}
\text{Table 2,} & B = 32 = & 2.9005 \\
 & H = 24 = & 1.6315 \\
\hline
 & & 4.5320 \\
 & & 20 = \tfrac{1}{2} \text{ length.} \\
\hline
 & & 90.6400 \\
\hline
\end{array}$$

770. *Fourth variety.*

$$\begin{array}{llr}
\text{Table 2,} & B \quad = 32 = & 2.9005 \\
 & H \quad = 24 = & 1.6315 \\
 & \text{Sum} = 56 = & 8.8829 \\
\hline
 & & 13.4149 \\
 & & 40 \\
\hline
 & & 6 \,|\, 536.5960 \\
\hline
 & & 89.432 \text{ gallons.} \\
\hline
\end{array}$$

TABLE I.

AREAS OF SEMI-SQUARES IN IMPERIAL GALLONS.

Sides in Inches.	0	1	2	3	4	5	6	7	8	9
		·00002	·00007	·00016	·00029	·00045	·00065	·00088	·00115	·00146
1	·0018	·0022	·0026	·0030	·0035	·0041	·0046	·0052	·0858	·0065
2	·0072	·0080	·0087	·0095	·0104	·0113	·0122	·0131	·0141	·0152
3	·0162	·0173	·0185	·0196	·0208	·0221	·0234	·0247	·0260	·0274
4	·0289	·0303	·0318	·0333	·0349	·0365	·0382	·0398	·0415	·0433
5	·0451	·0469	·0488	·0507	·0526	·0545	·0566	·0586	·0607	·0628
6	·0649	·0671	·0693	·0716	·0739	·0762	·0786	·0809	·0834	·0859
7	·0884	·0909	·0935	·0961	·0987	·1014	·1042	·1069	·1097	·1125
8	·1154	·1183	·1213	·1242	·1272	·1303	·1334	·1365	·1396	·1428
9	·1461	·1493	·1526	·1560	·1593	·1627	·1662	·1697	·1732	·1767
10	·1803	·1840	·1876	·1913	·1950	·1988	·2026	·2065	·2103	·2142
·11	·2182	·2222	·2262	·2303	·2344	·2385	·2426	·2468	·2511	·2554
12	·2597	·2640	·2684	·2728	·2773	·2818	·2863	·2908	·2954	·2001
13	·3048	·3095	·3142	·3190	·3238	·3286	·3335	·3385	·3434	·3484
14	·3534	·3585	·3636	·3688	·3739	·3791	·3844	·3897	·3950	·4003
15	·4057	·4112	·4166	·4221	·4277	·4332	·4388	·4445	·4502	·4559
16	·4616	·4674	·4733	·4791	·4850	·4909	·4969	·5029	·5090	·5150
17	·5211	·5273	·5335	·5397	·5460	·5523	·5586	·5649	·5713	·5778
18	·5843	·5908	·5973	·6039	·6105	·6172	·6239	·6306	·6373	·6441
19	·6510	·6579	·6648	·6717	·6787	·6857	·6927	·6998	·7070	·7141
20	·7213	·7285	·7358	·7431	·7504	·7578	·7652	·7727	·7802	·7877
21	·7952	·8028	·8105	·8181	·8258	·8336	·8413	·8491	·8570	·8649
22	·8728	·8807	·8887	·8967	·9048	·9129	·9210	·9292	·9374	·9457
23	·9539	·9622	·9706	·9790	·9874	·9959	1·0043	1·0129	1·0214	1·0300
24	1·0387	1·0474	1·0561	1·0648	1·0736	1·0824	1·0913	1·1002	1·1091	1·1180
25	1·1270	1·1361	1·1451	1·1543	1·1634	1·1726	1·1818	1·1910	1·2003	1·2097
26	1·2190	1·2284	1·2378	1·2473	1·2568	1·2663	1·2759	1·2855	1·2952	1·3049
27	1·3146	1·3243	1·3341	1·3440	1·3538	1·3637	1·3737	1·3836	1·3936	1·4037
28	1·4138	1·4239	1·4340	1·4442	1·4544	1·4647	1·4750	1·4853	1·4957	1·5061
29	1·5166	1·5270	1·5375	1·5481	1·5587	1·5693	1·5800	1·5906	1·6014	1·6121
30	1·6229	1·6338	1·6447	1·6556	1·6665	1·6775	1·6885	1·6996	1·7107	1·7218
31	1·7329	1·7441	1·7554	1·7666	1.7780	1·7893	1·8007	1·8121	1·8235	1·8350
32	1·8465	1·8581	1·8697	1·8813	1·8930	1·9047	1·9164	1·9282	1·9400	1·9519
33	1·9638	1·9757	1·9876	1·9996	2·0117	2·0237	2·0358	2·0480	2·0601	2·0723
34	2·0846	2·0969	2·1092	2·1215	2·1339	2·1463	2·1588	2·1713	2·1838	2·1964
35	2·2090	2·2216	2·2343	2·2470	2·2598	2·2726	2·2854	2·2983	2·3111	2·3241
36	2·3370	2·3500	2·3631	2·3762	2·3893	2·4024	2·4156	2·4288	2·4421	2.4554
37	2·4687	2·4820	2·4954	2·5089	2·5223	2·5358	2·5494	2·5630	2·5766	2·5902
38	2·6039	2·6176	2·6314	2·6452	2·6590	2·6729	2·6868	2·7007	2·7147	2·7287
39	2·7428	2·7569	2·7710	2·7851	2·7993	2·8136	2·8278	2·8421	2·8565	2·8708
40	2·8852	2·8997	2·9142	2·9287	2·9432	2·9578	2·9724	2·9871	3·0018	3·0165
41	3·0313	3·0461	3·0609	3·0758	3·0907	3·1057	3·1207	3·1357	3·1507	3·1658
42	3·1810	3·1961	3·2113	3·2266	3·2418	3·2572	3·2725	3·2879	3·3033	3·3188
43	3·3342	3·3498	3·3653	3·3809	3·3966	3·4122	3·4279	3·4437	3·4595	3·4753
44	3·4911	3·5070	3·5229	3·5389	3·5549	3·5709	3·5870	3·6031	3·6192	3·6354
45	3·6516	3·6679	3·6842	3·7005	3·7168	3·7332	3·7496	3·7661	3·7826	3·7991
46	3·8157	3·8323	3·8490	3·8657	3·8824	3·8991	3·9159	3·9327	3·9496	3·9665
47	3·9834	4·0004	4·0174	4·0344	4·0515	4·0686	4·0858	4·1030	4·1202	4·1374
48	4·1547	4·1721	4·1894	4·2068	4·2243	4·2417	4·2593	4·2768	4·2944	4·3120
49	4·3297	4·3473	4·3651	4·3828	4·4006	4·4185	4·4363	4·4542	4·4722	4·4902
50	4·5082	4·5262	4·5443	4·5624	4·5806	4·5988	4·6170	4·6353	4·6536	4·6719

TABLE I.—*continued.*

AREAS OF SEMI-SQUARES IN IMPERIAL GALLONS.

Sides in Inches	0	1	2	3	4	5	6	7	8	9
51	4·6903	4·7087	4·7272	4·7456	4·7642	4·7827	4·8013	4·8199	4·8386	4.8573
52	4·8760	4·8948	4·9136	4·9325	4·9513	4·9703	4·9892	4·0082	4·0272	5·0463
53	5·0654	5·0845	5·1037	5·1229	5·1421	5·1614	5·1807	5·2001	5·2195	5·2389
54	5·2583	5·2778	5·2974	5·3169	5·3365	5·3562	5·3758	5·3955	5·4153	5·4351
55	5·4549	5·4747	5·4946	5·5146	5·5345	5·5545	5·5746	5·5946	5·6147	5·6349
56	5·6551	5·6753	5·6955	5·7158	5·7361	5·7565	5·7769	5·7973	5·8178	5·8383
57	5·8588	5·8794	5·9000	5·9207	5·9413	5·9621	5·9828	6·0036	6·0244	6·0453
58	6·0662	6·0871	6·1081	6·1291	6·1502	6·1712	6·1924	6·2135	6·2347	6·2559
59	6·2772	6·2985	6·3198	6·3412	6·3626	6·3840	6·4055	6·4270	6·4486	6·4702
60	6·4918	6·5134	6·5351	6·5569	6·5786	6·6004	6·6223	6·6441	6·6660	6·6880
61	6·7100	6·7320	6·7540	6·7761	6·7983	6·8204	6·8426	6·8649	6·8871	6.9094
62	6·9318	6·9542	6·9766	6·9990	7·0215	7·0440	7·0666	7·0892	7·1118	7·1345
63	7·1572	7·1799	7·2027	7·2255	7·2484	7·2712	7·2942	7·3171	7·3401	7·3631
64	7·3862	7·4093	7·4324	7·4556	7·4788	7·5021	7·5253	7·5487	7·5720	7·5954
65	7·6188	7·6423	7·6658	7·6893	7·7129	7·7365	7·7601	7·7838	7·8075	7·8313
66	7·8550	7·8789	7·9027	7·9266	7·9505	7·9745	7·9985	8·0226	8·0466	8·0707
67	8·0949	8·1191	8·1433	8·1675	8·1918	8·2162	8·2405	8·2649	8·2893	8·3138
68	8·3383	8·3629	8·3874	8·4121	8·4367	8·4614	8·4861	8·5109	8·5357	8·5605
69	8·5854	8·6103	8·6352	8·6602	8·6852	8·7102	8·7353	8·7604	8·7856	8·8108
70	8·8360	8·8613	8·8866	8·9119	8·9373	8·9627	8·9881	9·0136	9·0391	9·0647
71	9·0903	9·1159	9·1416	9·1673	9·1930	9·2188	9·2446	9·2704	9·2963	9·3222
72	9·3482	9·3741	9·4002	9·4262	9·4523	9·4784	9·5046	9·5308	9·5570	9·5833
73	9·6096	9·6360	9·6624	9·6888	9·7152	9·7417	9·7682	9·7948	9·8214	9·8480
74	9·8747	9·9014	9·9282	9·9549	9·9818	10·0086	10·0355	10·0624	10·0894	10·1164
75	10·1434	10·1705	10·1976	10·2247	10·2519	10·2791	10·3063	10·3336	10·3609	10·3883
76	10·4157	10·4431	10·4706	10·4981	10·5256	10·5532	10·5808	10·6084	10·6361	10·6638
77	10·6916	10·7194	10·7472	10·7751	10·8030	10·8309	10·8589	10·8869	10·9149	10·9430
78	10·9711	10·9992	11·0274	11·0557	11·0839	11·1122	11·1405	11·1689	11·1973	11·2257
79	11·2542	11·2827	11·3113	11·3398	11·3685	11·3971	11·4258	11·4545	11·4833	11·5121
80	11·5409	11·5698	11·5987	11·6276	11·6566	11·6856	11·7147	11·7438	11·7729	11·8021
81	11·8313	11·8605	11·8898	11·9191	11·9484	11·9778	12·0072	12·0366	12·0661	12·0956
82	12·1252	12·1548	12·1844	12·2141	12·2438	12·2735	12·3033	12·3331	12·3629	12·3928
83	12·4227	12·4527	12·4827	12·5127	12·5428	12·5729	12·6030	12·6332	12·6634	12·6936
84	12·7239	12·7542	12·7845	12·8149	12·8453	12·8758	12·9063	12·9368	12·9674	12·9980
85	13·0286	13·0593	13·0900	13·1208	13·1515	13·1824	13·2132	13·2441	13·2750	13·3060
86	13·3370	13·3680	13·3991	13·4302	13·4613	13·4925	13·5237	13·5550	13·5863	13·6176
87	13·6490	13·6803	13·7118	13·7432	13·7747	13·8063	13·8379	13·8695	13·9011	13·9328
88	13·9645	13·9963	14·0281	14·0599	14·0918	14·1237	14·1556	14·1876	14·2196	14·2516
89	14·2837	14·3158	14·3480	14·3802	14·4124	14·4446	14·4769	14·5093	14·5416	14·5740
90	14·6065	14·6390	14·6715	14·7040	14·7366	14·7692	14·8019	14·8346	14·8673	14·9001
91	14·9329	14·9657	14·9986	15·0315	15·0644	15·0974	15·1304	15·1635	15·1966	15·2297
92	15·2629	15·2961	15·3293	15·3626	15·3959	15·4292	15·4626	15·4960	15·5295	15·5630
93	15·5965	15·6300	15·6636	15·6973	15·7309	15·7646	15·7984	15·8322	15·8660	15·8998
94	15·9337	15·9676	16·0016	16·0356	16·0696	16·1037	16·1378	16·1719	16·2061	16·2403
95	16·2745	16·3088	16·3431	16·3775	16·4119	16·4463	16·4807	16·5152	16·5498	16·5843
96	16·6189	16·6536	16·6883	16·7230	16·7577	16·7925	16·8273	16·8622	16·8971	16·9320
97	16·9670	17·0020	17·0370	17·0721	17·1072	17·1423	17·1775	17·2127	17·2480	17·2833
98	17·3186	17·3540	17·3894	17·4248	17·4603	17·4958	17·5313	17·5669	17·6025	17·6382
99	17·6739	17·7096	17·7453	17·7811	17·8170	17·8528	17·8887	17·9247	17·9606	17·9967
100	18·0327	18·0688	18·1049	18·1411	18·1773	18·2135	18·2497	18·2860	18·3224	18·3588

TABLE I.—*continued.*

AREAS OF SEMI-SQUARES IN IMPERIAL GALLONS.

Sides in Inches.	Area.	Sides in Inches.	Area.	Sides in Inches.	Area.	Sides in Inches.	Area.	Sides in Inches.	Area.
101	18·3952	151	41·1164	201	72·8539	251	113·6078	301	163·3781
102	18·7612	152	41·6628	202	73·5806	252	114·5149	302	164·4655
103	19·1309	153	42·2128	203	74·3110	253	115·4255	303	165·5565
104	19·5042	154	42·7664	204	75·0449	254	116·3398	304	166·6510
105	19·8811	155	43·3236	205	75·7824	255	117·2577	305	167·7492
106	20·2615	156	43·8844	206	76·5236	256	118·1791	306	168·8510
107	20·6456	157	44·4488	207	77·2683	257	119·1042	307	169·9564
108	21·0333	158	45·0168	208	78·0167	258	120·0329	308	171·0654
109	21·4247	159	45·5885	209	78·7687	259	120·9652	309	172·1781
110	21·8196	160	46·1637	210	79·5242	260	121·9011	310	173·2943
111	22·2181	161	46·7426	211	80·2834	261	122·8406	311	174·4141
112	22·6202	162	47·3250	212	81·0462	262	123·7837	312	175·5376
113	23·0260	163	47·9111	213	81·8126	263	124·7304	313	176·6646
114	23·4353	164	48·5008	214	82·5826	264	125·6807	314	177·7952
115	23·8483	165	49·0940	215	83·3562	265	126·6347	315	178·9295
116	24·2648	166	49·6909	216	84·1334	266	127·5922	316	180·0674
117	24·6850	167	50·2914	217	84·9142	267	128·5533	317	181·2088
118	25·1087	168	50·8955	218	85·6986	268	129·5181	318	182·3539
119	25·5361	169	51·5032	219	86·4867	269	130·4864	319	183·5026
120	25·9671	170	52·1145	220	87·2783	270	131·4584	320	184·6549
121	26·4017	171	52·7294	221	88·0735	271	132·4340	321	185·8108
122	26·8399	172	53·3480	222	88·8724	272	133·4132	322	186·9703
123	27·2817	173	53·9701	223	89·6748	273	134·3959	323	188·1334
124	27·7271	174	54·5958	224	90·4809	274	135·3823	324	189·3001
125	28·1761	175	55·2252	225	91·2906	275	136·3723	325	190·4704
126	28·6287	176	55·8581	226	92·1038	276	137·3659	326	191·6444
127	29·0849	177	56·4947	227	92·9207	277	138·3631	327	192·8219
128	29·5448	178	57·1348	228	93·7412	278	139·3640	328	194·0030
129	30·0082	179	57·7786	229	94·5653	279	140·3684	329	195·1878
130	30·4753	180	58·4260	230	95·3930	280	141·3764	330	196·3761
131	30·9459	181	59·0769	231	96·2243	281	142·3880	331	197·5681
132	31·4202	182	59·7315	232	97·0592	282	143·4033	332	198·7637
133	31·8981	183	60·3897	233	97·8977	283	144·4221	333	199·9629
134	32·3795	184	61·0515	234	98·7399	284	145·4446	334	201·1656
135	32·8646	185	61·7169	235	99·5856	285	146·4706	335	202·3720
136	33·3533	186	62·3859	236	100·4349	286	147·5003	336	203·5820
137	33·8456	187	63·0586	237	101·2879	287	148·5336	337	204·7956
138	34·3415	188	63·7348	238	102·1444	288	149·5705	338	206·0128
139	34·8410	189	64·4146	239	103·0046	289	150·6109	339	207·2336
140	35·3441	190	65·0981	240	103·8684	290	151·6550	340	208·4581
141	35·8508	191	65·7851	241	104·7257	291	152·7027	341	209·6861
142	36·3611	192	66·4758	242	105·6067	292	153·7540	342	210·9177
143	36·8751	193	67·1700	243	106·4813	293	154·8090	343	212·1530
144	37·3926	194	67·8679	244	107·3595	294	155·8675	344	213·3918
145	37·9138	195	68·5694	245	108·2413	295	156·9296	345	214·6343
146	38·4385	196	69·2744	246	109·1267	296	157·9953	346	215·8803
147	38·9669	197	69·9831	247	110·0157	297	159·0647	347	217·1300
148	39·4988	198	70·6954	248	110·9083	298	160·1376	348	218·3833
149	40·0344	199	71·4113	249	111·8046	299	161·2142	349	219·6401
150	40·5736	200	72·1308	250	112·7044	300	162·2943	350	220·9006

TABLE I.—*continued.*

AREAS OF SEMI-SQUARES IN IMPERIAL GALLONS.

Sides in Inches.	Area.	Sides in Inches.	Area.	Sides in Inches.	Area.	Sides in Inches.	Area.	Sides in Inches.	Area.
351	222·1647	401	289·9677	451	366·7870	501	452·6227	551	547·4747
352	223·4324	402	291·4157	452	368·4154	502	454·4314	552	549·4637
353	224·7037	403	292·8673	453	370·0473	503	456·2436	553	551·4563
354	225·9786	404	294·3226	454	371·6829	504	458·0595	554	553·4525
355	227·2572	405	295·7814	455	373·3221	505	459·8790	555	555·4524
356	228·5393	406	297·2439	456	374·9648	506	461·7021	556	557·4558
357	229·8250	407	298·7099	457	376·6112	507	463·5289	557	559·4628
358	231·1143	408	300·1796	458	378·2612	508	465·3592	558	561·4735
359	232·4073	409	301·6529	459	379·9148	509	467·1931	559	563·4877
360	233·7038	410	303·1298	460	381·5720	510	469·0306	560	565·5056
361	235·0040	411	304·6102	461	383·2328	511	470·8718	561	567·5271
362	236·3078	412	306·0943	462	384·8972	512	472·7165	562	569·5521
363	237·6151	413	307·5820	463	386·5653	513	474·5649	563	571·5808
364	238·9261	414	309·0733	464	388·2369	514	476·4168	564	573·6131
365	240·2407	415	310·5682	465	389·9121	515	478·2724	565	575·6490
366	241·5589	416	312·0668	466	391·5910	516	480·1316	566	577·6885
367	242·8807	417	313·5689	467	393·2734	517	481·9943	567	579·7316
368	244·2061	418	315·0746	468	394·9595	518	483·8607	568	581·7783
369	245·5351	419	316·5840	469	396·6492	519	485·7307	569	583·8286
370	246·8677	420	318·0969	470	398·3424	520	487·6043	570	585·8826
371	248·2039	421	319·6135	471	400·0393	521	489·4815	571	587·9401
372	249·5438	422	321·1336	472	401·7398	522	491·3623	572	590·0012
373	250·8872	423	322·6574	473	403·4439	523	493·2468	573	592·0660
374	252·2343	424	324·1847	474	405·1516	524	495·1348	574	594·1343
375	253·5849	425	325·7157	475	406·8629	525	497·0264	575	596·2063
376	254·9392	426	327·2503	476	408·5778	526	498·9216	576	598·2818
377	256·2970	427	328·7885	477	410·2963	527	500·8205	577	600·3610
378	257·6585	428	330·3303	478	412·0184	528	502·7229	578	602·4438
379	259·0236	429	331·8757	479	413·7442	529	504·6290	579	604·5302
380	260·3922	430	333·4247	480	415·4735	530	506·5387	580	606·6202
381	261·7645	431	334·9773	481	417·2064	531	508·4519	581	608·7138
382	263·1404	432	336·5335	482	418·9430	532	510·3688	582	610·8110
383	264·5199	433	338·0934	483	420·6832	533	512·2893	583	612·9118
384	265·9030	434	339·6568	484	422·4269	534	514·2134	584	615·0162
385	267·2898	435	341·2238	485	424·1743	535	516·1411	585	617·1242
386	268·6801	436	342·7945	486	425·9253	536	518·0724	586	619·2358
387	270·0740	437	344·3687	487	427·6798	537	520·0073	587	621·3511
388	271·4715	438	345·9466	488	429·4380	538	521·9458	588	623·4699
389	272·8727	439	347·5281	489	431·1998	539	523·8879	589	625·5924
390	274·2774	440	349·1132	490	432·9652	540	525·8337	590	627·7184
391	275·6858	441	350·7018	491	434·7342	541	527·7830	591	629·8481
392	277·0977	442	352·2941	492	436·5068	542	529·7359	592	631·9814
393	278·5133	443	353·8900	493	438·2831	543	531·6925	593	634·1182
394	279·9325	444	355·4895	494	440·0629	544	533·6526	594	636·2587
395	281·3553	445	357·0926	495	441·8463	545	535·6164	595	638·4028
396	282·7817	446	358·6993	496	443·6334	546	537·5838	596	640·5505
397	284·2116	447	360·3097	497	445·4240	547	539·5547	597	642·7018
398	285·6452	448	361·9236	498	447·2183	548	541·5293	598	644·8567
399	287·0825	449	363·5411	499	449·0161	549	543·5075	599	647·0152
400	288·5233	450	365·1623	500	450·8176	550	545·4893	600	649·1773

TABLE I.—*continued.*

AREAS OF SEMI-SQUARES IN IMPERIAL GALLONS.

Sides in Inches.	Area.	Sides in Inches.	Area.	Sides in Inches.	Area.	Sides in Inches.	Area.
601	651·3431	651	764·2278	701	886·1289	751	1017·0463
602	653·5124	652	766·5775	702	888·6589	752	1019·7566
603	655·6854	653	768·9307	703	891·1925	753	1022·4706
604	657·8619	654	771·2876	704	893·7297	754	1025·1881
605	660·0421	655	773·6481	705	896·2705	755	1027·9092
606	662·2258	656	776·0122	706	898·8149	756	1030·6340
607	664·4132	657	778·3799	707	901·3629	757	1033·3623
608	666·6042	658	780·7512	708	903·9145	758	1036·0943
609	668·7987	659	783·1261	709	906·4698	759	1038·8298
610	670·9969	660	785·5046	710	909·0286	760	1041·5690
611	673·1987	661	787·8867	711	911·5911	761	1044·3118
612	675·4041	662	790·2724	712	914·1571	762	1047·0581
613	677·6131	663	792·6618	713	916·7268	763	1049·8081
614	679·8257	664	795·0547	714	919·3000	764	1052·5617
615	682·0420	665	797·4513	715	921·8769	765	1055·3189
616	684·2618	666	799·8514	716	924·4574	766	1058·0797
617	686·4852	667	802·2552	717	927·0415	767	1060·8441
618	688·7122	668	804·6625	718	929·6292	768	1063·6122
619	690·9429	669	807·0735	719	932·2205	769	1066·3838
620	693·1771	670	809·4881	720	934·8154	770	1069·1590
621	695·4150	671	811·9063	721	937·4139	771	1071·9379
622	697·6565	672	814·3281	722	940·0160	772	1074·7203
623	699·9015	673	816·7535	723	942·6217	773	1077·5064
624	702·1502	674	819·1825	724	945·2311	774	1080·2960
625	704·4025	675	821·6151	725	947·8440	775	1083·0893
626	706·6584	676	824·0513	726	950·4606	776	1085·8862
627	708·9179	677	826·4911	727	953·0807	777	1088·6866
628	711·1810	678	828·9346	728	955·7045	778	1091·4907
629	713·4477	679	831·3816	729	958·3318	779	1094·2984
630	715·7180	680	833·8322	730	960·9628	780	1097·1097
631	717·9920	681	836·2865	731	963·5974	781	1099·9246
632	720·2695	682	838·7443	732	966·2356	782	1102·7431
633	722·5506	683	841·2058	733	968·8774	783	1105·5653
634	724·8354	684	843·6709	734	971·5228	784	1108·3910
635	727·1237	685	846·1396	735	974·1718	785	1111·2203
636	729·4157	686	848·6118	736	976·8244	786	1114·0532
637	731·7112	687	851·0877	737	979·4806	787	1116·8898
638	734·0104	688	853·5672	738	982·1404	788	1119·7299
639	736·3132	689	856·0503	739	984·8038	789	1122·5737
640	738·6196	690	858·5370	740	987·4709	790	1125·4211
641	740·9295	691	861·0273	741	990·1415	791	1128·2720
642	743·2431	692	863·5213	742	992·8158	792	1131·1266
643	745·5603	693	866·0188	743	995·4936	793	1133·9848
644	747·8812	694	868·5199	744	998·1751	794	1136·8466
645	750·2056	695	871·0247	745	1000·8602	795	1139·7120
646	752·5336	696	873·5330	746	1003·5488	796	1142·5810
647	754·8652	697	876·0450	747	1006·2411	797	1145·4536
648	757·2005	698	878·5606	748	1008·9370	798	1148·3298
649	759·5393	699	881·0797	749	1011·6365	799	1151·2096
650	761·8817	700	883·6025	750	1014·3396	800	1154·0931

TABLE II.

AREAS OF CIRCLES IN IMPERIAL GALLONS.

Diameters in Inches.	0	1	2	3	4	5	6	7	8	9
1	·0028	·0034	·0040	·0047	·0055	·0063	·0072	·0081	·0091	·0102
2	·0113	·0124	·0137	·0149	·0163	·0177	·0191	·0206	·0222	·0238
3	·0254	·0272	·0290	·0308	·0327	·0346	·0367	·0387	·0409	·0430
4	·0453	·0476	·0499	·0523	·0548	·0573	·0599	·0625	·0652	·0680
5	·0708	·0736	·0765	·0795	·0825	·0856	·0888	·0920	·0952	·0986
6	·1019	·1053	·1088	·1124	·1160	·1196	·1233	·1271	·1309	·1348
7	·1387	·1427	·1468	·1509	·1551	·1593	·1636	·1679	·1723	·1767
8	·1812	·1858	·1904	·1951	·1998	·2046	·2094	·2143	·2193	·2243
9	·2294	·2345	·2397	·2449	·2502	·2556	·2610	·2665	·2720	·2776
10	·2832	·2889	·2947	·3005	·3063	·3122	·3182	·3243	·3303	·3365
11	·3427	·3490	·3553	·3616	·3681	·3746	·3811	·3877	·3944	·4011
12	·4078	·4147	·4215	·4285	·4355	·4425	·4496	·4568	·4640	·4713
13	·4787	·4860	·4935	·5010	·5086	·5162	·5239	·5316	·5394	·5472
14	·5551	·5631	·5711	·5792	·5873	·5955	·6037	·6120	·6204	·6288
15	·6373	·6458	·6544	·6630	·6717	·6805	·6893	·6982	·7071	·7161
16	·7251	·7342	·7433	·7525	·7618	·7711	·7805	·7899	·7994	·8090
17	·8186	·8282	·8379	·8477	·8575	·8674	·8774	·8874	·8974	·9075
18	·9177	·9279	·9382	·9485	·9589	·9694	·9799	·9905	1·0011	1·0118
19	1·0225	1·0333	1·0441	1·0551	1·0660	1·0770	1·0881	1·0992	1·1104	1·1217
20	1·1330	1·1443	1·1558	1·1672	1·1788	1·1903	1·2020	1·2137	1·2254	1·2372
21	1·2491	1·2610	1·2730	1·2851	1·2972	1·3093	1·3215	1·3338	1·3461	1·3585
22	1·3709	1·3834	1·3960	1·4086	1·4212	1·4339	1·4467	1·4595	1·4724	1·4854
23	1·4984	1·5114	1·5246	1·5377	1·5510	1·5642	1·5776	1·5910	1·6044	1·6179
24	1·6315	1·6451	1·6588	1·6726	1·6863	1·7002	1·7141	1·7281	1·7421	1·7562
25	1·7703	1·7845	1·7987	1·8131	1·8274	1·8418	1·8563	1·8708	1·8854	1·9001
26	1·9148	1·9295	1·9443	1·9592	1·9741	1·9891	2·0042	2·0193	2·0344	2·0496
27	2·0649	2·0802	2·0956	2·1110	2·1265	2·1421	2·1577	2·1734	2·1891	2·2049
28	2·2207	2·2366	2·2525	2·2685	2·2846	2·3007	2·3169	2·3331	2·3494	2·3657
29	2·3821	2·3986	2·4151	2·4317	2·4483	2·4650	2·4817	2·4985	2·5154	2·5323
30	2·5493	2·5663	2·5834	2·6005	2·6177	2·6349	2·6523	2·6696	2·6870	2·7045
31	2·7221	2·7396	2·7573	2·7750	2·7928	2·8106	2·8284	2·8464	2·8644	2·8824
32	2·9005	2·9187	2·9369	2·9551	2·9735	2·9919	3·0103	3·0288	3·0473	3·0660
33	3·0847	3·1033	3·1221	3·1410	3·1599	3·1788	3·1978	3·2169	3·2360	3·2552
34	3·2744	3·2937	3·3130	3·3324	3·3519	3·3714	3·3910	3·4106	3·4303	3·4500
35	3·4698	3·4897	3·5096	3·5296	3·5496	3·5697	3·5898	3·6100	3·6303	3·6506
36	3·6710	3·6914	3·7119	3·7324	3·7530	3·7736	3·7943	3·8151	3·8359	3·8568
37	3·8777	3·8987	3·9198	3·9409	3·9620	3·9833	4·0045	4·0259	4·0472	4·0687
38	4·0902	4·1117	4·1334	4·1550	4·1767	4·1985	4·2204	4·2423	4·2642	4·2862
39	4·3083	4·3304	4·3526	4·3748	4·3971	4·4195	4·4419	4·4643	4·4869	4·5094
40	4·5321	4·5548	4·5775	4·6003	4·6232	4·6461	4·6690	4·6921	4·7152	4·7383
41	4·7615	4·7848	4·8081	4·8314	4·8549	4·8783	4·9019	4·9255	4·9491	4·9728
42	4·9966	5·0204	5·0443	5·0682	5·0922	5·1163	5·1404	5·1645	5·1888	5·2130
43	5·2374	5·2618	5·2862	5·3107	5·3353	5·3599	5·3846	5·4093	5·4341	5·4589
44	5·4838	5·5088	5·5338	5·5588	5 5840	5·6091	5·6344	5·6597	5·6850	5·7104
45	5·7359	5·7614	5·7870	5·8126	5·8383	5·8641	5·8899	5·9157	5·9417	5·9676
46	5·9937	6·0198	6·0459	6·0721	6·0984	6·1247	6·1510	6·1775	6·2040	6·2305
47	6·2571	6·2838	6·3105	6·3372	6·3641	6·3909	6·4179	6·4449	6·4719	6·4990
48	6·5262	6·5534	6·5807	6·6080	6·6354	6·6620	6·6904	6·7179	6·7455	6·7732
49	6·8010	6·8287	6·8566	6·8845	6·9124	6·9405	6·9685	6·9967	7·0248	7·0531
50	7·0814	7·1097	7·1381	7·1666	7·1951	7·2237	7·2524	7·2810	7·3098	7·3386

TABLE II.—*continued.*

AREAS OF CIRCLES IN IMPERIAL GALLONS.

Diameters in Inches.	0	1	2	3	4	5	6	7	8	9
51	7·3675	7·3964	7·4254	7·4544	7·4835	7·5126	7·5418	7·5711	7·6004	7·6298
52	7·6592	7·6887	7·7183	7·7479	7·7775	7·8072	7·8370	7·8668	7·8967	7·9266
53	7·9566	7·9867	8·0168	8·0470	8·0772	8·1075	8·1378	8·1682	8·1987	8·2292
54	8·2597	8·2903	8·3210	8·3518	8·3825	8·4134	8·4443	8·4753	8·5063	8·5373
55	8·5685	8·5997	8·6309	8·6622	8·6936	8·7250	8·7564	8·7880	8·8196	8·8512
56	8·8829	8·9146	8·9465	8·9783	9·0102	9·0422	9·0743	9·1064	9·1385	9·1707
57	9·2030	9·2353	9·2677	9·3001	9·3326	9·3651	9·3977	9·4304	9·4631	9·4959
58	9·5287	9·5616	9·5945	9·6275	9·6606	9·6937	9·7269	9·7601	9·7934	9·8267
59	9·8601	9·8936	9·9271	9·9607	9·9943	10·0280	10·0617	10·0955	10·1293	10·1632
60	10·1972	10·2312	10·2653	10·2994	10·3336	10·3679	10·4022	10·4365	10·4709	10·5054
61	10·5399	10·5745	10·6092	10·6439	10·6786	10·7134	10·7483	10·7832	10·8182	10·8533
62	10·8884	10·9235	10·9587	10·9940	11·0293	11·0647	11·1001	11·1356	11·1712	11·2068
63	11·2424	11·2781	11·3139	11·3497	11·3856	11·4216	11·4576	11·4936	11·5298	11·5659
64	11·6022	11·6384	11·6748	11·7112	11·7476	11·7842	11·8207	11·8573	11·8940	11·9308
65	11·9676	12·0044	12·0413	12·0783	12·1153	12·1524	12·1895	12·2267	12·2640	12·3013
66	12·3386	12·3760	12·4135	12·4511	12·4886	12·5263	12·5640	12·6017	12·6396	12·6774
67	12·7154	12·7533	12·7914	12·8295	12·8676	12·9058	12·9441	12·9824	13·0208	13·0593
68	13·0978	13·1363	13·1749	13·2136	13·2523	13·2911	13·3299	13·3688	13·4078	13·4468
69	13·4858	13·5249	13·5641	13·6033	13·6426	13·6820	13·7214	13·7608	13·8003	13·8399
70	13·8795	13·9192	13·9590	13·9988	14·0386	14·0785	14·1185	14·1585	14·1986	14·2387
71	14·2789	14·3192	14·3595	14·3999	14·4403	14·4808	14·5213	14·5619	14·6025	14·6432
72	14·6840	14·7248	14·7657	14·8066	14·8476	14·8886	14·9297	14·9709	15·0121	15·0534
73	15·0947	15·1361	15·1775	15·2190	15·2606	15·3022	15·3439	15·3856	15·4274	15·4692
74	15·5111	15·5531	15·5951	15·6371	15·6792	15·7214	15·7637	15·8059	15·8483	15·8907
75	15·9332	15·9757	16·0182	16·0609	16·1036	16·1463	16·1891	16·2320	16·2749	16·3179
76	16·3609	16·4040	16·4471	16·4903	16·5336	16·5769	16·6202	16·6637	16·7071	16·7507
77	16·7943	16·8379	16·8816	16·9254	16·9692	17·0131	17·0570	17·1010	17·1450	17·1891
78	17·2333	17·2775	17·3218	17·3661	17·4105	17·4550	17·4995	17·5440	17·5886	17·6333
79	17·6780	17·7228	17·7676	17·8125	17·8575	17·9025	17·9476	17·9927	18·0379	18·0831
80	18·1284	18·1738	18·2192	18·2646	18·3101	18·3557	18·4013	18·4470	18·4928	18·5386
81	18·5844	18·6304	18·6763	18·7224	18·7684	18·8146	18·8608	18·9070	18·9534	18·9997
82	19·0462	19·0926	19·1392	19·1858	19·2324	19·2791	19·3259	19·3727	19·4196	19·4665
83	19·5135	19·5606	19·6077	19·6548	19·7021	19·7493	19·7967	19·8441	19·8915	19·9390
84	19·9866	20·0342	20·0819	20·1296	20·1774	20·2252	20·2731	20·3211	20·3691	20·4171
85	20·4653	20·5135	20·5617	20·6100	20·6583	20·7067	20·7552	20·8037	20·8523	20·9009
86	20·9496	20·9984	21·0472	21·0961	21·1450	20·1940	21·2430	21·2921	21·3412	21·3904
87	21·4397	21·4890	21·5384	21·5878	21·6373	21·6868	21·7364	21·7861	21·8358	21·8856
88	21·9354	21·9853	22·0352	22·0852	22·1352	22·1854	22·2355	22·2857	22·3360	22·3863
89	22·4367	22·4872	22·5377	22·5883	22·6389	22·6895	22·7403	22·7911	22·8419	22·8928
90	22·9438	22·9948	23·0459	23·0970	23·1482	23·1994	23·2507	23·3021	23·3535	23·4049
91	23·4565	23·5080	23·5597	23·6114	23·6631	23·7149	23·7668	23·8187	23·8707	23·9227
92	23·9748	24·0270	24·0792	24·1314	24·1838	24·2361	24·2886	24·3411	24·3936	24·4462
93	24·4989	24·5516	24·6043	24·6572	24·7100	24·7630	24·8160	24·8690	24·9222	24·9753
94	25·0285	25·0818	25·1352	25·1886	25·2420	25·2955	25·3491	25·4027	25·4564	25·5101
95	25·5639	25·6177	25·6717	25·7256	25·7796	25·8337	25·8878	25·9420	25·9963	26·0506
96	26·1049	26·1593	26·2138	26·2683	26·3229	26·3776	26·4323	26·4870	26·5418	26·5967
97	26·6516	26·7066	26·7616	26·8167	26·8719	26·9271	26·9823	27·0377	27·0930	27·1485
98	27·2040	27·2595	27·3151	27·3708	27·4265	27·4823	27·5381	27·5940	27·6499	27·7059
99	27·7620	27·8181	27·8743	27·9305	27·9868	28·0431	28·0995	28·1560	28·2125	28·2690
100	28·3257	28·3823	28·4391	28·4959	28·5527	28·6096	28·6666	28·7236	28·7807	28·8378

TABLE II.—*continued.*

AREAS OF CIRCLES IN IMPERIAL GALLONS.

Diameters in Inches.	0	1	2	3	4	5	6	7	8	9
101	28·8950	28·9522	29·0096	29·0669	29·1243	29·1818	29·2393	29·2969	29·3546	29·4123
102	29·4700	29·5278	29·5857	29·6436	29·7016	29·7596	29·8177	29·8759	29·9341	29·9924
103	30·0507	30·1091	30·1675	30·2260	30·2845	30·3432	30·4018	30·4605	30·5193	30·5781
104	30·6370	30·6960	30·7550	30·8140	30·8732	30·9323	30·9916	31·0508	31·1102	31·1696
105	31·2290	31·2886	31·3481	31·4077	31·4674	31·5272	31·5870	31·6468	31·7067	31·7667
106	31·8267	31·8868	31·9469	32·0071	32·0674	32·1277	32·1880	32·2485	32·3089	32·3695
107	32·4301	32·4907	32·5514	32·6122	32·6730	32·7338	32·7948	32·8558	32·9168	32·9779
108	33·0391	33·1003	33·1615	33·2229	33·2842	33·3457	33·4072	33·4687	33·5303	33·5920
109	33·6537	33·7155	33·7773	33·8392	33·9012	33·9632	34·0252	34·0874	34·1495	34·2118
110	34·2741	34·3364	34·3988	34·4613	34·5238	34·5863	34·6490	34·7117	34·7744	34·8372
111	34·9001	34·9630	35·0259	35·0890	35·1520	35·2152	35·2784	35·3416	35·4049	35·4683
112	35·5317	35·5952	35·6587	35·7223	35·7860	35·8497	35·9134	35·9772	36·0411	36·1051
113	36·1690	36·2331	36·2972	36·3613	36·4256	36·4898	36·5542	36·6185	36·6830	36·7475
114	36·8120	36·8766	36·9413	37·0060	37·0708	37·1357	37·2006	37·2655	37·3305	37·3956
115	37·4607	37·5259	37·5911	37·6564	37·7217	37·7871	37·8526	37·9181	37·9837	38·0493
116	38·1150	38·1808	38·2466	38·3124	38·3783	38·4443	38·5103	38·5764	38·6426	38·7088
117	38·7750	38·8413	38·9077	38·9741	39·0406	39·1071	39·1737	39·2404	39·3071	39·3738
118	39·4407	39·5075	39·5745	39·6415	39·7085	39·7756	39·8428	39·9100	39·9773	40·0446
119	40·1120	40·1794	40·2469	40·3145	40·3821	40·4498	40·5175	40·5853	40·6531	40·7210
120	40·7890	40·8570	40·9250	40·9932	41·0613	41·1296	41·1979	41·2662	41·3346	41·4031
121	41·4716	41·5402	41·6088	41·6775	41·7463	41·8151	41·8839	41·9528	42·0218	42·0908
122	42·1599	42·2291	42·2983	42·3675	42·4368	42·5062	42·5756	42·6451	42·7147	42·7843
123	42·8539	42·9236	42·9934	43·0632	43·1331	43·2030	43·2730	43·3431	43·4132	43·4833
124	43·5536	43·6238	43·6942	43·7646	43·8350	43·9055	43·9761	44·0467	44·1173	44·1881
125	44·2589	44·3297	44·4006	44·4716	44·5426	44·6136	44·6848	44·7560	44·8272	44·8985
126	44·9698	45·0412	45·1127	45·1842	45·2558	45·3275	45·3991	45·4709	45·5427	45·6146
127	45·6865	45·7585	45·8305	45·9026	45·9747	46·0469	46·1192	46·1915	46·2639	46·3363
128	46·4088	46·4813	46·5539	46·6266	46·6993	46·7721	46·8449	46·9178	46·9907	47·0637
129	47·1368	47·2099	47·2830	47·3563	47·4295	47·5029	47·5763	47·6497	47·7232	47·7968
130	47·8704	47·9441	48·0178	48·0916	48·1654	48·2393	48·3133	48·3873	48·4614	48·5355
131	48·6097	48·6839	48·7582	48·8326	48·9070	48·9815	49·0560	49·1306	49·2052	49·2799
132	49·3547	49·4295	49·5043	49·5793	49·6542	49·7293	49·8044	49·8795	49·9547	50·0300
133	50·1053	50·1807	50·2561	50·3316	50·4071	50·4827	50·5584	50·6341	50·7099	50·7857
134	50·8616	50·9375	51·0135	51·0896	51·1657	51·2419	51·3181	51·3944	51·4707	51·5471
135	51·6235	51·7001	51·7766	51·8532	51·9299	52·0067	52·0834	52·1603	52·2372	52·3142
136	52·3912	52·4682	52·5454	52·6226	52·6998	52·7771	52·8545	52·9319	53·0094	53·0869
137	53·1645	53·2421	53·3198	53·3976	53·4754	53·5532	53·6312	53·7091	53·7872	53·8653
138	53·9434	54·0216	54·0999	54·1782	54·2566	54·3350	54·4135	54·4921	54·5707	54·6493
139	54·7280	54·8068	54·8856	54·9645	55·0435	55·1225	55·2015	55·2807	55·3598	55·4390
140	55·5183	55·5977	55·6771	55·7565	55·8360	55·9156	55·9952	56·0749	56·1546	56·2344
141	56·3143	56·3942	56·4742	56·5542	56·6343	56·7144	56·7946	56·8748	56·9551	57·0355
142	57·1159	57·1964	57·2769	57·3575	57·4381	57·5188	57·5996	57·6804	57·7613	57·8422
143	57·9232	58·0042	58·0853	58·1665	58·2477	58·3290	58·4103	58·4917	58·5731	58·6546
144	58·7361	58·8177	58·8994	58·9811	59·0629	59·1447	59·2266	59·3086	59·3906	59·4726
145	59·5547	59·6369	59·7191	59·8014	59·8838	59·9662	60·0486	60·1311	60·2137	60·2963
146	60·3790	60·4618	60·5446	60·6274	60·7103	60·7933	60·8763	60·9594	61·0425	61·1257
147	61·2090	61·2923	61·3756	61·4591	61·5425	61·6261	61·7097	61·7933	61·8770	61·9608
148	62·0446	62·1284	62·2124	62·2964	62·3804	62·4645	62·5487	62·6329	62·7171	62·8015
149	62·8858	62·9703	63·0548	63·1393	63·2239	63·3086	63·3933	63·4781	63·5629	63·6478
150	63·7328	63·8178	63·9029	63·9880	64·0731	64·1584	64·2437	64·3290	64·4144	64·4999

TABLE II.—*continued.*

AREAS OF CIRCLES IN IMPERIAL GALLONS.

Diameters in Inches.	0	1	2	3	4	5	6	7	8	9
151	64·5854	64·6710	64·7566	64·8423	64·9280	65·0138	65·0997	65·1856	65·2716	65·3576
152	65·4437	65·5298	65·6160	65·7022	65·7886	65·8749	65·9613	66·0478	66·1344	66·2209
153	66·3076	66·3943	66·4811	66·5679	66·6548	66·7417	66·8287	66·9157	67·0028	67·0900
154	67·1772	67·2645	67·3518	67·4392	67·5266	67·6141	67·7017	67·7893	67·8770	67·9647
155	68·0525	68·1403	68·2282	68·3161	68·4041	68·4922	68·5803	68·6685	68·7567	68·8450
156	68·9334	69·0218	69·1103	69·1988	69·2873	69·3760	69·4647	69·5534	69·6422	69·7311
157	69·8200	69·9090	69·9980	70·0871	70·1762	70·2654	70·3547	70·4440	70·5333	70·6228
158	70·7122	70·8018	70·8914	70·9810	71·0707	71·1605	71·2503	71·3402	71·4301	71·5201
159	71·6102	71·7003	71·7904	71·8806	71·9709	72·0613	72·1516	72·2421	72·3326	72·4231
160	72·5138	72·6044	72·6952	72·7859	72·8768	72·9677	73·0586	73·1496	73·2407	73·3318
161	73·4230	73·5142	73·6055	73·6969	73·7883	73·8798	73·9713	74·0629	74·1545	74·2462
162	74·3379	74·4297	74·5216	74·6135	74·7055	74·7975	74·8896	74·9817	75·0739	75·1662
163	75·2585	75·3509	75·4433	75·5358	75·6283	75·7209	75·8136	75·9063	75·9991	76·0919
164	76·1848	76·2777	76·3707	76·4637	76·5569	76·6500	76·7432	76·8365	76·9298	77·0232
165	77·1167	77·2102	77·3037	77·3974	77·4910	77·5848	77·6786	77·7724	77·8663	77·9603
166	78·0543	78·1483	78·2425	78·3366	78·4309	78·5252	78·6195	78·7139	78·8084	78·9029
167	78·9975	79·0921	79·1868	79·2816	79·3764	79·4718	79·5662	79·6612	79·7562	79·8513
168	79·9464	80·0416	80·1369	80·2322	80·3276	80·4230	80·5185	80·6140	80·7096	80·8053
169	80·9010	80·9968	81·0926	81·1885	81·2844	81·3804	81·4765	81·5726	81·6687	81·7650
170	81·8612	81·9576	82·0540	82·1504	82·2469	82·3435	82·4401	82·5368	82·6335	82·7303
171	82·8271	82·9240	83·0210	83·1180	83·2151	83·3122	83·4094	83·5067	83·6039	83·7013
172	83·7987	83·8962	83·9937	84·0913	84·1889	84·2866	84·3844	84·4822	84·5801	84·6780
173	84·7760	84·8740	84·9721	85·0702	85·1684	85·2667	85·3650	85·4634	85·5618	85·6603
174	85·7589	85·8575	85·9561	86·0548	86·1536	86·2524	86·3513	86·4503	86·5493	86·6483
175	86·7474	86·8466	86·9458	87·0451	87·1444	87·2438	87·3433	87·4428	87·5424	87·6420
176	87·7417	87·8414	87·9412	88·0410	88·1409	88·2409	88·3409	88·4410	88·5411	88·6413
177	88·7416	88·8419	88·9422	89·0426	89·1431	89·2436	89·3442	89·4449	89·5455	89·6463
178	89·7471	89·8480	89·9489	90·0499	90·1509	90·2520	90·3532	90·4544	90·5556	90·6570
179	90·7583	90·8598	90·9613	91·0628	91·1644	91·2661	91·3678	91·4696	91·5714	91·6733
180	91·7752	91·8772	91·9793	92·0814	92·1836	92·2858	92·3881	92·4904	92·5928	92·6953
181	92·7978	92·9004	93·0030	93·1057	93·2084	93·3112	93·4140	93·5170	93·6199	93·7229
182	93·8260	93·9292	94·0323	94·1356	94·2389	94·3423	94·4457	94·5491	94·6527	94·7563
183	94·8599	94·9636	95·0674	95·1712	95·2750	95·3790	95·4830	95·5870	95·6911	95·7952
184	95·8995	96·0037	96·1080	96·2124	96·3169	96·4214	96·5259	96·6305	96·7352	96·8399
185	96·9447	97·0495	97·1544	97·2593	97·3644	97·4694	97·5745	97·6797	97·7849	97·8902
186	97·9956	98·1010	98·2064	98·3119	98·4175	98·5231	98·6288	98·7345	98·8403	98·9462
187	99·0521	99·1581	99·2641	99·3702	99·4763	99·5825	99·6888	99·7951	99·9014	100·007
188	100·114	100·220	100·327	100·434	100·540	100·647	100·754	100·861	100·968	101·075
189	101·182	101·289	101·396	101·503	101·610	101·718	101·825	101·933	102·040	102·148
190	102·255	102·363	102·471	102·578	102·686	102·794	102·902	103·010	103·118	103·226
191	103·335	103·443	103·551	103·659	103·768	103·876	103·985	104·093	104·202	104·311
192	104·419	104·528	104·637	104·746	104·855	104·964	105·073	105·182	105·291	105·401
193	105·510	105·619	105·729	105·838	105·948	106·057	106·167	106·277	106·386	106·496
194	106·606	106·716	106·826	106·936	107·046	107·156	107·267	107·377	107·487	107·598
195	107·708	107·818	107·929	108·040	108·150	108·261	108·372	108·483	108·594	108·705
196	108·816	108·927	109·038	109·149	109·260	109·371	109·483	109·594	109·706	109·817
197	109·929	110·040	110·152	110·264	110·376	110.487	110·599	110·711	110·823	110·935
198	111·048	111·160	111·272	111·384	111·497	111·609	111·722	111·834	111·947	112·059
199	112·172	112·285	112·398	112·511	112.624	112·737	112·850	112·963	113·076	113·189
200	113·302	113·416	113·529	113·642	113·756	113·870	113·983	114·097	114·211	114·324

TABLE II.—*continued.*

AREAS OF CIRCLES IN IMPERIAL GALLONS.

Diameters in Inches.	0	1	2	3	4	5	6	7	8	9
201	114·438	114·552	114·666	114·780	114·894	115·008	115·122	115·237	115·351	115·465
202	115·580	115·694	115·809	115·923	116·038	116·153	116·267	116·382	116·497	116·612
203	116·727	116·842	116·957	117·072	117·187	117·303	117·418	117·533	117·649	117·764
204	117·880	117·995	118·111	118·227	118·342	118·458	118·574	118·690	118·806	118·922
205	119·038	119·154	119·271	119·387	119·503	119·620	119·736	119·853	119·969	120·086
206	120·202	120·319	120·436	120·553	120·670	120·787	120·904	121·021	121·138	121·255
207	121·372	121·490	121·607	121·724	121·842	121·959	122·077	122·195	122·312	122·430
208	122·548	122·666	122·784	122·902	123·020	123·138	123·256	123·374	123·492	123·611
209	123·729	123·847	123·966	124·084	124·203	124·322	124·440	124·559	124·678	124·797
210	124·916	125·035	125·154	125·273	125·392	125·511	125·631	125·750	125·869	125·989
211	126·108	126·228	126·348	126·467	126·587	126·707	126·827	126·947	127·066	127·186
212	127·307	127·427	127·547	127·667	127·787	127·908	128·028	128·149	128·269	128·390
213	128·510	128·631	128·752	128·873	128·994	129·114	129·235	129·356	129·478	129·599
214	129·720	129·841	129·962	130·084	130·205	130·327	130·448	130·570	130·692	130·813
215	130·935	131·057	131·179	131·301	131·423	131·545	131·667	131·789	131·911	132·034
216	132·156	132·278	132·401	132·523	132·646	132·768	132·891	133·014	133·137	133·260
217	133·382	133·505	133·628	133·751	133·875	133·998	134·121	134·244	134·368	134·491
218	134·615	134·738	134·862	134·985	135·109	135·233	135·357	135·480	135·604	135·728
219	135·852	135·977	136·101	136·225	136·349	136·473	136·598	136·722	136·847	136·971
220	137·096	137·221	137·345	137·470	137·595	137·720	137·845	137·970	138·095	138·220
221	138·345	138·470	138·596	138·721	138·846	138·972	139·097	139·223	139·348	139·474
222	139·600	139·726	139·852	139·977	140·103	140·229	140·356	140·482	140·608	140·734
223	140·860	140·987	141·113	141·240	141·366	141·493	141·619	141·746	141·873	142·000
224	142·127	142·253	142·380	142·508	142·635	142·762	142·889	143·016	143·144	143·271
225	143·398	143·526	143·653	143·781	143·909	144·036	144·164	144·292	144·420	144·548
226	144·676	144·804	144·932	145·060	145·188	145·317	145·445	145·573	145·702	145·830
227	145·959	146·088	146·216	146·345	146·474	146·603	146·732	146·861	146·990	147·119
228	147·248	147·377	147·506	147·636	147·765	147·894	148·024	148·153	148·283	148·413
229	148·542	148·672	148·802	148·932	149·062	149·192	149·322	149·452	149·582	149·712
230	149·842	149·973	150·103	150·234	150·364	150·495	150·625	150·756	150·887	151·017
231	151·148	151·279	151·410	151·541	151·672	151·803	151·935	152·066	152·197	152·328
232	152·460	152·591	152·723	152·854	152·986	153·118	153·249	153·381	153·513	153·645
233	153·777	153·909	154·041	154·173	154·305	154·438	154·570	154·702	154·835	154·967
234	155·100	155·232	155·365	155·498	155·630	155·763	155·896	156·029	156·162	156·295
235	156·428	156·561	156·695	156·828	156·961	157·095	157·228	157·362	157·495	157·629
236	157·762	157·896	158·030	158·164	158·298	158·432	158·566	158·700	158·834	158·968
237	159·102	159·236	159·371	159·505	159·640	159·774	159·909	160·043	160·178	160·313
238	160·448	160·582	160·717	160·852	160·987	161·122	161·258	161·393	161·528	161·663
239	161·799	161·934	162·070	162·205	162·341	162·476	162·612	162·748	162·884	163·020
240	163·156	163·292	163·428	163·564	163·700	163·836	163·972	164·109	164·245	164·382
241	164·518	164·655	164·791	164·928	165·065	165·201	165·338	165·475	165·612	165·749
242	165·886	166·023	166·160	166·298	166·435	166·572	166·710	166·847	166·985	167·122
243	167·260	167·398	167·535	167·673	167·811	167·949	168·087	168·225	168·363	168·501
244	168·639	168·778	168·916	169·054	169·193	169·331	169·470	169·608	169·747	169·886
245	170·025	170·163	170·302	170·441	170·580	170·719	170·858	170·998	171·137	171·276
246	171·415	171·555	171·694	171·834	171·973	172·113	172·253	172·392	172·532	172·672
247	172·812	172·952	173·092	173·232	173·372	173·512	173·652	173·793	173·933	174·073
248	174·214	174·354	174·495	174·636	174·776	174·917	175·058	175·199	175·340	175·481
249	175·622	175·763	175·904	176·045	176·186	176·328	176·469	176·611	176·752	176·894
250	177·035	177·177	177·319	177·460	177·602	177·744	177·886	178·028	178·170	178·312

TABLE II.—*continued.*

AREAS OF CIRCLES IN IMPERIAL GALLONS.

Diameters in Inches.	0	1	2	3	4	5	6	7	8	9
251	178·454	178·596	178·739	178·881	179·024	179·166	179·308	179·451	179·594	179·736
252	179·879	180·022	180·165	180·308	180·451	180·594	180·737	180·880	181·023	181·166
253	181·310	181·453	181·596	181·740	181·883	182·027	182·170	182·314	182·458	182·602
254	182·746	182·890	183·034	183·178	183·322	183·466	183·610	183·754	183·899	184·043
255	184·187	184·332	184·476	184·621	184·766	184·910	185·055	185·200	185·345	185·490
256	185·635	185·780	186·925	186·070	186·215	186·361	186·506	186·651	186·797	186·942
257	187·088	187·234	187·379	187·525	187·671	187·817	187·963	188·108	188·255	188·401
258	188·547	188·693	188·839	188·985	189·132	189·278	189·425	189·571	189·718	189·864
259	190·011	190·158	190·305	190·452	190·599	190·746	190·893	191·040	191·187	191·334
260	191·481	191·629	191·776	191·923	192·071	192·218	192·366	192·514	192·661	192·809
261	192·957	193·105	193·253	193·401	193·549	193·697	193·845	193·993	194·142	194·290
262	194·438	194·587	194·735	194·884	195·033	195·181	195·330	195·479	195·628	195·777
263	195·926	196·075	196·224	196·373	196·522	196·671	196·821	196·970	197·119	197·269
264	197·418	197·568	197·718	197·867	198·017	198·167	198·317	198·467	198·617	198·767
265	198·917	199·067	199·217	199·367	199·518	199·668	199·819	199·969	200·120	200·270
266	200·421	200·572	200·722	200·873	201·024	201·175	201·326	201·477	201·628	201·779
267	201·931	202·082	202·233	202·385	202·536	202·688	202·839	202·991	203·143	203·294
268	203·446	203·598	203·750	203·902	204·054	204·206	204·358	204·510	204·662	204·815
269	204·967	205·120	205·272	205·425	205·577	205·730	205·883	206·035	206·188	206·341
270	206·494	206·647	206·800	206·953	207·106	207·259	207·413	207·566	207·719	207·873
271	208·026	208·180	208·333	208·487	208·641	208·795	208·948	109·102	209·256	209·410
272	209·564	209·719	209·873	210·027	210·181	210·336	210·490	210·644	210·799	210·954
273	211·108	211·263	211·418	211·572	211·727	211·882	212·037	212·192	212·347	212·502
274	212·658	212·813	212·968	213·123	213·279	213·434	213·590	213·746	213·901	214·057
275	214·213	214·368	214·524	214·680	214·836	214·992	215·148	215·305	215·461	215·617
276	215·773	215·930	216·086	216·243	216·399	216·556	216·713	216·869	217·026	217·183
277	217·340	217·497	217·654	217·811	217·968	218·125	218·282	218·440	218·597	218·754
278	218·912	219·069	219·227	219·385	219·542	219·700	219·858	220·016	220·174	220·332
279	220·490	220·648	220·806	220·964	221·122	221·281	221·439	221·597	221·756	221·914
280	222·073	222·232	222·390	222·549	222·708	222·867	223·026	223·185	223·344	223·503
281	223·662	223·821	223·981	224·140	224·299	224·459	224·618	224·778	224·937	225·097
282	225·257	225·417	225·576	225·736	225·896	226·056	226·216	226·377	226·537	226·697
283	226·857	227·018	227·178	227·338	227·499	227·660	227·820	227·981	228·142	228·302
284	228·463	228·624	228·785	228·946	229·107	229·268	229·430	229·591	229·752	229·914
285	230·075	230·237	230·398	230·560	230·721	230·883	231·045	231·207	231·369	231·530
286	231·692	231·854	232·017	332·179	232·341	232·503	232·666	232·828	232·990	233·153
287	233·316	233·478	233·641	233·804	233·966	234·129	234·292	234·455	234·618	234·781
288	234·944	235·107	235·271	235·434	235·597	235·761	235·924	236·088	236·251	236·415
289	236·579	236·742	236·906	237·070	237·234	237·398	237·562	237·726	237·890	238·054
290	238·219	238·383	238·547	238·712	238·876	249·041	239·205	239·370	239·535	239·700
291	239·864	240·029	240·194	240·359	240·524	240·689	240·855	241·020	241·185	241·350
292	241·516	241·681	241·847	242·012	242·178	242·344	242·509	242·675	242·841	243·007
293	243·173	243·339	243·505	243·671	243·837	244·003	244·170	244·336	244·503	244·669
294	244·836	245·002	245·169	245·335	245·502	245·669	245·836	246·003	246·170	246·337
295	246·504	246·671	246·838	247·006	247·173	247·340	247·508	247·675	247·843	248·010
296	248·178	248·346	248·513	248·681	248·849	249·017	249·185	249·353	249·521	249·689
297	249·858	250·026	250·194	250·363	250·531	250·700	250·868	251·037	251·206	251·374
298	251·543	251·712	251·881	252·050	252·219	252·388	252·557	252·726	252·895	253·065
299	253·234	253·404	253·573	253·743	253·912	254·082	254·251	254·421	254·591	254·761
300	254·931	255·101	255·271	255·441	255·611	255·781	255·952	256·122	256·292	256·463

TABLE III.

TWO-THIRD AREAS OF CIRCLES IN IMPERIAL GALLONS.

Inch.	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9
12	·2719	·2765	·2811	·2857	·2904	·2951	·2998	·3046	·3094	·3142
13	·3191	·3241	·3290	·3340	·3391	·3442	·3493	·3544	·3596	·3649
14	·3701	·3754	·3808	·3862	·3916	·3970	·4025	·4081	·4136	·4192
15	·4249	·4306	·4363	·4421	·4478	·4537	·4596	·4655	·4714	·4774
16	·4834	·4895	·4956	·5017	·5079	·5141	·5204	·5267	·5330	·5393
17	·5457	·5522	·5587	·5652	·5717	·5783	·5849	·5916	·5983	·6051
18	·6118	·6187	·6255	·6324	·6393	·6463	·6533	·6603	·6674	·6745
19	·6817	·6889	·6961	·7034	·7107	·7181	·7254	·7329	·7403	·7478
20	·7554	·7629	·7705	·7782	·7859	·7936	·8014	·8092	·8170	·8249
21	·8328	·8407	·8487	·8567	·8648	·8729	·8810	·8892	·8974	·9057
22	·9140	·9223	·9307	·9391	·9475	·9560	·9645	·9731	·9817	·9903
23	·9990	1·0077	1·0164	1·0252	1·0340	1·0429	1·0518	1·0607	1·0697	1·0787
24	1·0877	1·0968	1·1059	1·1151	1·1243	1·1335	1·1428	1·1521	1·1614	1·1708
25	1·1802	1·1897	1·1992	1·2087	1·2183	1·2279	1·2376	1·2473	1·2570	1·2667
26	1·2765	1·2864	1·2963	1·3062	1·3161	1·3261	1·3361	1·3462	1·3563	1·3665
27	1·3766	1·3868	1·3971	1·4074	1·4177	1·4281	1·4385	1·4489	1·4594	1·4699
28	1·4805	1·4911	1·5017	1·5124	1·5231	1·5338	1·5446	1·5554	1·5663	1·5772
29	1·5881	1·5991	1·6101	1·6212	1·6322	1·6434	1·6545	1·6657	1·6770	1·6882
30	1·6995	1·7109	1·7223	1·7337	1·7452	1·7567	1·7682	1·7798	1·7914	1·8030
31	1·8147	1·8265	1·8382	1·8500	1·8619	1·8737	1·8857	1·8976	1·9096	1·9216
32	1·9337	1·9458	1·9579	1·9701	1·9823	1·9946	2·0069	2·0192	2·0316	2·0440
33	2·0564	2·0689	2·0814	2·0940	2·1066	2·1192	2·1319	2·1446	2·1574	2·1701
34	2·1830	2·1958	2·2087	2·2217	2·2346	2·2476	2·2607	2·2738	2·2869	2·3001
35	2·3133	2·3265	2·3398	2·3531	2·3664	2·3798	2·3933	2·4067	2·4202	2·4338
36	2·4473	2·4610	2·4746	2·4883	2·5020	2·5158	2·5296	2·5434	2·5573	2·5712
37	2·5852	2·5992	2·6132	2·6273	2·6414	2·6555	2·6697	2·6839	2·6982	2·7125
38	2·7268	2·7412	2·7556	2·7700	2·7845	2·7991	2·8136	2·8282	2·8428	2·8575
39	2·8722	2·8870	2·9018	2·9166	2·9314	2·9463	2·9613	2·9763	2·9913	3·0063
40	3·0214	3·0365	3·0517	3·0669	3·0821	3·0974	3·1127	3·1281	3·1435	3·1589
41	3·1744	3·1899	3·2054	3·2210	3·2366	3·2523	3·2680	3·2837	3·2995	3·3153
42	3·3311	3·3470	3·3629	3·3789	3·3949	3·4109	3·4270	3·4431	3·4592	3·4754
43	3·4916	3·5079	3·5242	3·5405	3·5569	3·5733	3·5897	3·6062	3·6227	3·6393
44	3·6559	3·6725	3·6892	3·7059	3·7227	3·7395	3·7563	3·7732	3·7901	3·8070
45	3·8240	3·8410	3·8580	3·8751	3·8923	3·9094	3·9266	3·9439	3·9611	3·9785
46	3·9958	4·0132	4·0306	4·0481	4·0656	4·0832	4·1007	4·1183	4·1360	4·1537
47	4·1714	4·1892	4·2070	4·2249	4·2427	4·2607	4·2786	4·2966	4·3146	4·3327
48	4·3508	4·3690	4·3872	4·4054	4·4236	4·4419	4·4603	4·4787	4·4971	4·5155
49	4·5340	4·5525	4·5711	4·5897	4·6083	4·6270	4·6457	4·6645	4·6833	4·7021
50	4·7210	4·7399	4·7588	4·7778	4·7968	4·8158	4·8349	4·8541	4·8732	4·8924

TABLE V.

TWO-FIFTEENTH AREAS OF

Inch.	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9
1	·00037	·00045	·00054	·00063	·00074	·00084	·00096	·00109	·00122	·00136
2	·00151	·00166	·00183	·00199	·00217	·00236	·00255	·00275	·00296	·00317
3	·00339	·00362	·00386	·00411	·00436	·00462	·00489	·00517	·00545	·00574
4	·00604	·00634	·00666	·00698	·00731	·00764	·00799	·00834	·00870	·00906
5	·00944	·00982	·01021	·01060	·01101	·01142	·01184	·01227	·01270	·01314
6	·01359	·01405	·01451	·01499	·01547	·01595	·01645	·01695	·01746	·01798
7	·01850	·01903	·01957	·02012	·02068	·02124	·02181	·02239	·02297	·02357
8	·02417	·02478	·02539	·02601	·02665	·02728	·02793	·02858	·02924	·02991

TABLE IV.

ONE-THIRD AREAS OF CIRCLES IN IMPERIAL GALLONS.

Inch	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9
10	·0944	·0963	·0982	·1001	·1021	·1041	·1061	·1081	·1101	·1122
11	·1142	·1163	·1184	·1205	·1227	·1248	·1271	·1293	·1315	·1337
12	·1360	·1382	·1405	·1428	·1452	·1475	·1499	·1523	·1547	·1571
13	·1596	·1620	·1645	·1670	·1695	·1721	·1746	·1772	·1798	·1824
14	·1851	·1877	·1904	·1931	·1958	·1985	·2013	·2040	·2068	·2096
15	·2124	·2153	·2181	·2210	·2239	·2268	·2298	·2327	·2357	·2387
16	·2417	·2447	·2478	·2509	·2539	·2571	·2602	·2633	·2665	·2697
17	·2729	·2761	·2793	·2826	·2859	·2892	·2925	·2958	·2992	·3025
18	·3059	·3093	·3128	·3166	·3197	·3231	·3267	·3302	·3337	·3373
19	·3409	·3445	·3481	·3517	·3554	·3590	·3627	·3664	·3702	·3739
20	·3777	·3815	·3853	·3891	·3929	·3968	·4007	·4046	·4085	·4124
21	·4164	·4204	·4244	·4284	·4324	·4365	·4405	·4446	·4487	·4528
22	·4570	·4612	·4653	·4695	·4738	·4780	·4823	·4865	·4908	·4951
23	·4995	·5038	·5082	·5126	·5170	·5214	·5260	·5303	·5348	·5393
24	·5439	·5484	·5530	·5575	·5621	·5668	·5714	·5760	·5807	·5854
25	·5901	·5948	·5996	·6044	·6092	·6140	·6188	·6236	·6285	·6334
26	·6383	·6432	·6481	·6531	·6581	·6631	·6681	·6731	·6782	·6832
27	·6883	·6934	·6986	·7037	·7089	·7140	·7192	·7245	·7297	·7350
28	·7402	·7455	·7509	·7562	·7615	·7669	·7723	·7777	·7831	·7886
29	·7941	·7996	·8051	·8106	·8161	·8217	·8273	·8329	·8385	·8441
30	·8498	·8554	·8611	·8669	·8726	·8783	·8841	·8899	·8957	·9015
31	·9074	·9132	·9191	·9250	·9309	·9368	·9428	·9488	·9548	·9608
32	·9669	·9729	·9790	·9851	·9912	·9973	1·0034	1·0096	1·0158	1·0220
33	1·0282	1·0345	1·0407	1·0470	1·0533	1·0596	1·0660	1·0723	1·0787	1·0851
34	1·0915	1·0979	1·1044	1·1108	1·1173	1·1238	1·1303	1·1369	1·1435	1·1500
35	1·1566	1·1633	1·1699	1·1765	1·1832	1·1899	1·1966	1·2034	1·2101	1·2169
36	1·2237	1·2305	1·2373	1·2442	1·2510	1·2579	1·2648	1·2717	1·2737	1·2856
37	1·2926	1·2996	1·3066	1·3136	1·3207	1·3278	1·3349	1·3420	1·3491	1·3562
38	1·3634	1·3706	1·3778	1·3850	1·3923	1·3995	1·4068	1·4141	1·4214	1·4288
39	1·4361	1·4435	1·4509	1·4583	1·4657	1·4732	1·4806	1·4881	1·4956	1·5032
40	1·5107	1·5183	1·5258	1·5335	1·5411	1·5487	1·5564	1·5640	1·5717	1·5795
41	1·5872	1·5949	1·6027	1·6105	1·6183	1·6261	1·6340	1·6418	1·6497	1·6576
42	1·6656	1·6735	1·6815	1·6894	1·6974	1·7054	1·7135	1·7215	1·7296	1·7377
43	1·7458	1·7539	1·7621	1·7703	1·7784	1·7866	1·7949	1·8031	1·8114	1·8197
44	1·8280	1·8363	1·8446	1·8530	1·8613	1·8697	1·8781	1·8866	1·8950	1·9035
45	1·9120	1·9205	1·9290	1·9376	1·9461	1·9547	1·9633	1·9719	1·9806	1·9892
46	1·9979	2·0066	2·0153	2·0241	2·0328	2·0416	2·0504	2·0592	2·0680	2·0769
47	2·0857	2·0946	2·1035	2·1124	2·1214	2·1303	2·1393	2·1483	2·1573	2·1664
48	2·1754	2·1845	2·1936	2·2027	2·2118	2·2210	2·2301	2·2393	2·2485	2·2578

TABLE V.—*continued.*

CIRCLES IN IMPERIAL GALLONS.

Inch.	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9
9	·03059	·03127	·03196	·03266	·03337	·03408	·03480	·03553	·03627	·03701
10	·03777	·03852	·03929	·04007	·04085	·04164	·04243	·04324	·04405	·04487
11	·04570	·04653	·04737	·04822	·04908	·05004	·05082	·05170	·05259	·05348
12	·05436	·05529	·05621	·05714	·05807	·05901	·05996	·06091	·06188	·06285
13	·06383	·06481	·06581	·06681	·06781	·06883	·06985	·07089	·07192	·07297
14	·07393	·07509	·07615	·07723	·07831	·07941	·08051	·08161	·08273	·08385
15	·08498	·08611	·08726	·08841	·08957	·09074	·09191	·09309	·09428	·09548

TABLE VI.

NET CISTERN AND COUCH BUSHELS.

Gros Bushels.	Net Bus.	Gross Bushels.	Net Bus.	Gross Bushels.	Net Bus.	Gross Bushels.	Net Bus.	Gross Bushels	Net Bus.	Gross Bushels.	Net Bus.	Gross Bushels.	Net Bus.
·1	·0815	42	34·23	92	74·98	142	115·73	192	156·48	242	197·23	292	237·98
·2	·163	43	35·04	93	75·79	143	116·54	193	157·29	243	198·04	293	238·79
·3	·244	44	35·86	94	76·61	144	117·36	194	158·11	244	198·86	294	239·61
·4	·326	45	36·67	95	77·42	145	118·17	195	158·92	245	199·67	295	240·42
·5	·407	46	37·49	96	78·24	146	118·99	196	159·74	246	200·49	296	241·24
·6	·489	47	38·30	97	79·05	147	119·80	197	160·55	247	201·30	297	242·05
·7	·570	48	39·12	98	79·87	148	120·62	198	161·37	248	202·12	298	242·87
·8	·652	49	39·93	99	80·68	149	121·43	199	162·18	249	202·93	299	243·68
·9	·733	50	40·75	100	81·50	150	122·25	200	163·00	250	203·75	300	244·50
1	·815	51	41·56	101	82·31	151	123·06	201	163·81	251	204·56	310	252·65
2	1·63	52	42·38	102	83·13	152	123·88	202	164·63	252	205·38	320	260·80
3	2·44	53	43·19	103	83·94	153	124·69	203	165·44	253	206·19	330	268·95
4	3·26	54	44·01	104	84·76	154	125·51	204	166·26	254	207·01	340	277·10
5	4·07	55	44·82	105	85·57	155	126·32	205	167·07	255	207·82	350	285·25
6	4·89	56	45·64	106	86·39	156	127·14	206	167·89	256	208·64	360	293·40
7	5·70	57	46·45	107	87·20	157	127·95	207	168·70	257	209·45	370	301·55
8	6·52	53	47·27	108	88·02	158	128·77	208	169·52	258	210·27	380	309·70
9	7·33	59	48·08	109	88·83	159	129·58	209	170·33	259	211·08	390	317·85
10	8·15	60	48·90	110	89·65	160	130·40	210	171·15	260	211·90	400	326·00
11	8·96	61	49·71	111	90·46	161	131·21	211	171·96	261	212·71	410	334·15
12	9·78	62	50·53	112	91·28	162	132·03	212	172·78	262	213·53	420	342·30
13	10·59	63	51·34	113	92·09	163	132·84	213	173·59	263	214·34	430	350·45
14	11·41	64	52·16	114	92·91	164	133·66	214	174·41	264	215·16	440	358·60
15	12·22	65	52·97	115	93·72	165	134·47	215	175·22	265	215·97	450	366·75
16	13·04	66	53·79	116	94·54	166	135·29	216	176·04	266	216·79	460	374·90
17	13·85	67	54·60	117	95·35	167	136·10	217	176·85	267	217·60	470	383·05
18	14·67	68	55·42	118	96·17	168	136·92	218	177·67	268	218·42	480	391·20
19	15·48	69	56·23	119	96·98	169	137·73	219	178·48	269	219·23	490	399·35
20	16·30	70	57·05	120	97·80	170	138·55	220	179·30	270	220·05	500	407·50
21	17·11	71	57·86	121	98·61	171	139·36	221	180·11	271	220·86	510	415·65
22	17·93	72	58·68	122	99·43	172	140·18	222	180·93	272	221·68	520	423·80
23	18·74	73	59·49	123	100·24	173	140·99	223	181·74	273	222·49	530	431·95
24	19·56	74	60·31	124	101·06	174	141·81	224	182·56	274	223·31	540	440·10
25	20·37	75	61·12	125	101·87	175	142·62	225	183·37	275	224·12	550	448·25
26	21·19	76	61·94	126	102·69	176	143·44	226	184·19	276	224·94	560	456·40
27	22·00	77	62·75	127	103·50	177	144·25	227	185·00	277	225·75	570	464·55
28	22·82	78	63·57	128	104·32	178	145·07	228	185·82	278	226·57	580	472·70
29	23·63	79	64·38	129	105·13	179	145·88	229	186·63	279	227·38	590	480·85
30	24·45	80	65·20	130	105·95	180	146·70	230	187·45	280	228·20	600	489·00
31	25·26	81	66·01	131	106·76	181	147·51	231	188·26	281	229·01	610	497·15
32	26·08	82	66·83	132	107·58	182	148·33	232	189·08	282	229·83	620	505·30
33	26·89	83	67·64	133	108·39	183	149·14	233	189·89	283	230·64	630	513·45
34	27·71	84	68·46	134	109·21	184	149·96	234	190·71	284	231·46	640	521·60
35	28·62	85	69·27	135	110·02	185	150·77	235	191·52	285	232·27	650	529·75
36	29·34	86	70·09	136	110·84	186	151·59	236	192·34	286	233·09	675	550·12
37	30·15	87	70·90	137	111·65	187	152·40	237	193·15	287	233·90	700	570·50
38	30·97	88	71·72	138	112·47	188	153·22	238	193·97	288	234·72	800	652·00
39	31·78	89	72·53	139	113·28	189	154·03	239	194·78	289	235·53	900	733·50
40	32·60	90	73·35	140	114·10	190	154·85	240	195·60	290	236·35	1000	815·00
41	33·41	91	74·16	141	114·91	191	155·66	241	196·41	291	237·16		

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